1. Answer the following questions.
   (a) (3 points) Does a BIBD exist for $a = 4, k = 2, r = 2$ and $b = 4$ experiment? Explain.

   (b) (3 points) Complete the following ANOVA table for CRD and RCBD for the general case with p treatments and q experimental units per treatment. Consider precisely two treatments and either 14 replicates per treatment (completely randomized) or 14 blocks (block design).

Table 1: RCBD and CRD comparision

	RCBD		CRD	
Source	general df	specific df	general df	specific df
Treatments				
Blocks				
Residual				
Total				

   (c) (3 points) Discuss the difference between RCBD and CRD. Explain the advantages of using RCBD instead of CRD.

2. The tensile strength of portland cement is being studied. Four different mixing techniques can be used economically. The following data have been collected:
   (a) (3 points) Test the hypothesis that mixing techniques affect the strength of the cement. Use $\alpha = 0.05$ and draw appropriate conclusions.

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Analysis of Variance Table
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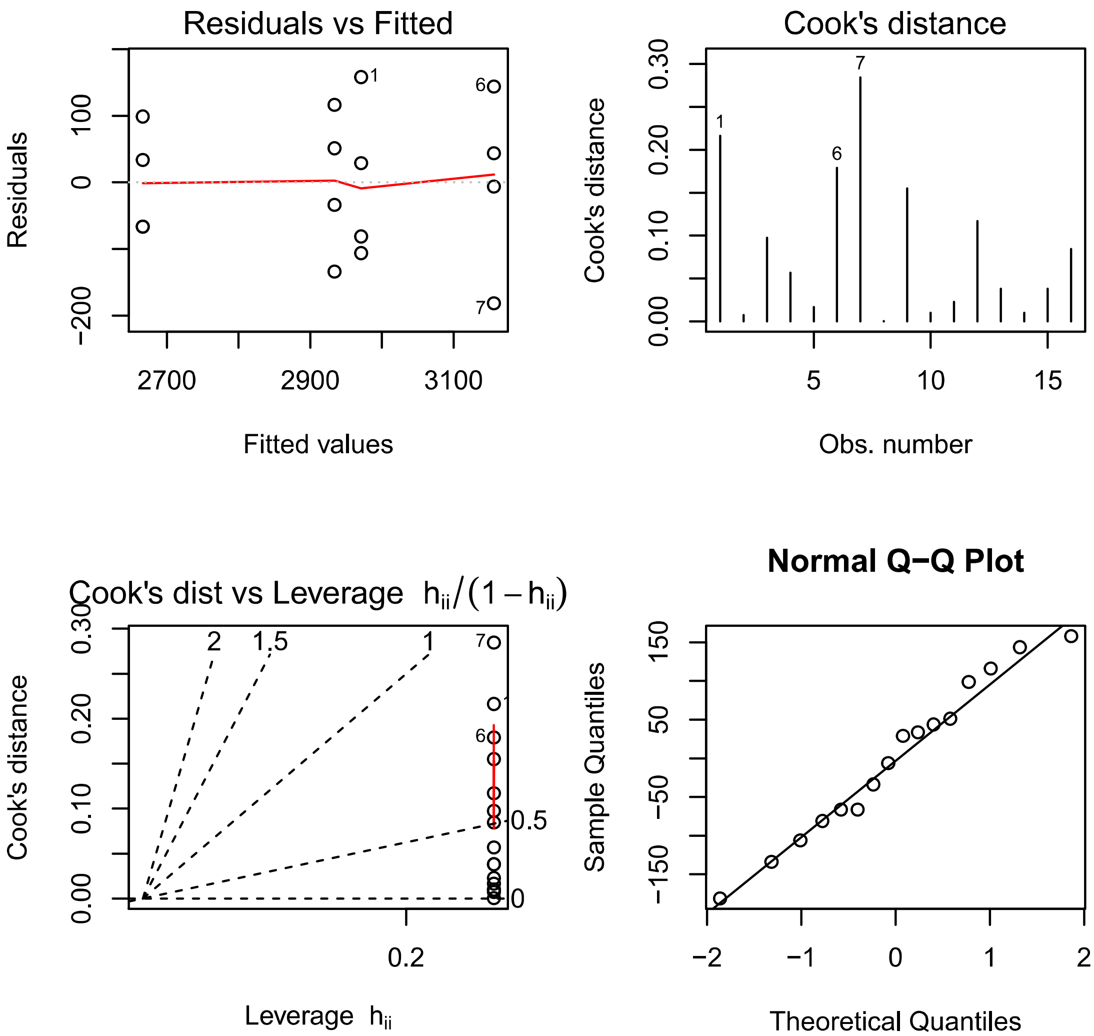
Mixing Technique	Tensile Strength (lb/in^2)			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

```
Response: Y
Df Sum Sq Mean Sq F value    Pr(>F)
factor(Treatment)  3 489740  163247  12.728 0.0004887 ***
Residuals         12 153908   12826
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
2.5 %    97.5 %
(Intercept)        2847.62412 3094.3759
factor(Treatment)2   10.77016  359.7298
factor(Treatment)3 -211.72984  137.2298
factor(Treatment)4 -479.22984 -130.2702
```

(b) (3 points) Analyze the residuals from this experiment. What conclusion would you draw about the validity of the normality assumptions?



3. **Latin Square Design:**

(a) (2 points) Give a 4×4 Latin square design using the treatments, say A, B, C and D occur in the first row and first column in alphabetical order.

(b) (6 points) A courier company is interested in deciding between five brands (D, P, F, C and R) of cars for its subsequent purchase of fleet cars. The brands are all comparable in the purchase price. The company wants to conduct a study that will enable them to compare the brands concerning operating costs. For this purpose, they select five drivers (Rows). In addition, the study will be carried out over five weeks (Columns = weeks). Each week a driver is assigned to a car using randomization and a Latin Square Design. The average cost per mile is recorded at the end of each week and is tabulated below:

		Week				
		1	2	3	4	5
Drivers	1	5.83 D	6.22 P	7.67 F	9.43 C	6.57 R
	2	4.80 P	7.56 D	10.34 C	5.82 R	9.86 F
	3	7.43 F	11.29 C	7.01 R	10.48 D	9.27 P
	4	6.60 R	9.54 F	11.11 D	10.84 P	15.05 C
	5	11.24 C	6.34 R	11.30 P	12.58 F	16.04 D

Analyze the data from this experiment ($\alpha = 0.05$) draw appropriate conclusions.

4. **Graeco-Latin square:**

(a) (2 points) How does Latin square design differ from Graeco-Latin square design?

(b) (6 points) An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a colour television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. Suppose the engineer suspects that the workplaces used by the four operators may represent an additional source of variation. A fourth factor, workplace (α, β, γ, δ), may be introduced, and an experiment was conducted, yielding the Graeco-Latin square that follows. Analyze the data from this experiment ($\alpha = 0.05$) and draw conclusions.

	Operator			
Order of Assembly	1	2	3	4
1	Cβ = 11	Bγ = 10	Dδ = 14	Aα = 8
2	Bα = 8	Cδ =12	Aγ = 10	Dβ = 12
3	Aδ = 9	Dα = 11	Bβ = 7	Cγ = 15
4	Dγ = 9	Aβ = 8	Cα = 18	Bδ = 6

5. In an experiment to compare the quality of four different brands of magnetic recording tape, five 2400-ft reels of each brand (A-D) was selected, and the number of flaws in each reel was determined.

A	B	C	D
10	14	13	17
5	12	18	16
12	17	10	12
14	9	15	22
8	8	18	14

Find the least-squares estimates of μ, α_1, α_2, α_3, α_4, μ_1, μ_2, μ_3 and μ_4 using the CRD in matrix form. Note that $\mu_i = \mu + \alpha_i,\ i = 1, 2, 3, 4$.

(a) (6 points) Use the constraint: $\alpha_4 = 0$.

(b) (5 points) Use the constraint: $\alpha_1 = 0$.

6. **Balanced incomplete block design:**

   (a) (2 points) What is the difference between randomised complete block design and balanced incomplete block design?

   (b) (4 points) An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test, he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow.

	car1	car2	car3	car4	car5	$y_{i\cdot}$
add1		17	14	13	12	56
add2	14	14		13	10	51
add3	12		13	12	9	46
add4	13	11	11	12		47
add5	11	12	10		8	41
y_j	50	54	48	50	39	

Calculate the adjusted means:

$$\hat{\mu}_i = \hat{\mu} + \hat{\tau}_i, \qquad i = 1, 2, 3, 4, 5.$$

7. An article in Industrial Quality Control (1956, pp. 5-8) describes an experiment to investigate the effect of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are as follows:

	Glass Type	
Phosphor Type	1	2
1	280, 290, 285	240, 235, 240
2	300, 310, 295	260, 240, 235
3	290, 285, 290	220, 225, 230

(a) (7 points) Is there any indication that either factor influences brightness? Use $\alpha = 0.05$.

(b) (2 points) Do the two factors interact? Use $\alpha = 0.05$.

8. Consider a two-factor factorial model with random effects of the following type:

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \begin{cases} i = 1, \cdots, a \\ j = 1, \cdots, b \\ k = 1, \cdots, r \end{cases} \tag{1}$$

and where $\alpha_i,\ \beta_j,\ (\alpha\beta)_{ij}$ are random variables independent of each other and of ϵ_{ijk} (i.e., $\{\alpha_i\} \perp \{\beta_j\} \perp \{(\alpha\beta)_{ij}\} \perp \{\epsilon_{ijk}\}$). Assume that the random variables $\alpha_i,\ \beta_j,\ (\alpha\beta)_{ij}$ and ϵ_{ijk} are normally distributed with mean zero and variances given by $\text{Var}(\alpha_i) = \sigma_\alpha^2,\ \text{Var}(\beta_j) = \sigma_\beta^2,\ \text{Var}((\alpha\beta)_{ij}) = \sigma_{\alpha\beta}^2$, and $\text{Var}(\epsilon_{ijk}) = \sigma^2$; with $\alpha = \frac{1}{a}\sum_{i=1}^{a}\alpha_i,\ \beta = \frac{1}{b}\sum_{j=1}^{b}\beta_j,\ (\alpha\beta)_{.j} = \frac{1}{a}\sum_{i=1}^{a}(\alpha\beta)_{ij}$, and $(\alpha\beta)_{..} = \frac{1}{ab}\sum\sum(\alpha\beta)_{ij}$.

(a) (2 points) Determine $\mathbb{E}(y_{ijk})$.

(b) (2 points) Determine $\text{Var}(y_{ijk})$.

(c) (5 points) Show that

$$\text{Cov}(y_{ijk}, y_{i'j'k'}) = \begin{cases} \sigma^2_\alpha + \sigma^2_\beta + \sigma^2_{\alpha\beta} + \sigma^2 & i = i',\ j = j',\ k = k' \\ \sigma^2_\alpha + \sigma^2_\beta + \sigma^2_{\alpha\beta} & i = i',\ j = j',\ k \neq k' \\ \sigma^2_\alpha & i = i',\ j \neq j',\ \forall k, k' \\ \sigma^2_\beta & i \neq i',\ j = j',\ \forall k, k' \\ 0 & i \neq i',\ j \neq j',\ \forall k, k' \end{cases}$$

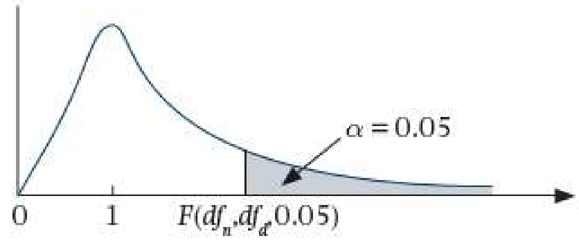
Critical Values for Student's t Distributions

	α						
ν	**.10**	**.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	4.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.618	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

TABLE 4 (a)

F distribution ($\alpha = 0.05$)

The entries in this table are critical values of F for which the area under the curve to the right is equal to 0.05.



	df_1									
df_2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93