

Question 1 [16 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Classify the following function

(1)

$$f(x) = x^6(9x^4 - 4).$$

- (a) Polynomial function
- (b) Power function
- (c) Trigonometric function
- (d) Rational function
- (e) Exponential function

1.2 Evaluate the limit, if it exist.

(1)

$$\lim_{x \rightarrow \infty} \frac{x^7 - 5}{x^6 + 10}$$

- (a) $-\infty$
- (b) 0
- (c) ∞
- (d) $\frac{1}{5}$
- (e) $-\frac{1}{5}$

1.3 Find the function $f \circ g$ and its domain if $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{x}{x+5}$.

(2)

- (a) $\frac{x-1}{x+5}$, $D = (-\infty, -5) \cup (-5, 0) \cup (0, \infty)$
- (b) $\frac{x-1}{x+5}$, $D = (-\infty, -5) \cup (-5, \infty)$
- (c) $-\frac{5}{x}$, $D = (-\infty, -5) \cup (-5, 0) \cup (0, \infty)$

(d) $-\frac{5}{x}, \quad D = (-\infty, 0) \cup (0, \infty)$

(e) None of the above

1.4 Differentiate $g(t) = t^5 \cos t$ (2)

(a) $g'(x) = t^4 \cos t - t^5 \sin t$

(b) $g'(x) = t^4 \cos t + t^5 \sin t$

(c) $g'(x) = 5t^4 \cos t + t^5 \sin t$

(d) $g'(x) = 5t^4 \cos t - t^5 \sin t$

(e) $g'(x) = 5t^4 \cos t - 5t^4 \sin t$

1.5 Find the domain of the following function (2)

$$f(x) = \frac{1 - e^{x^2}}{1 - e^{36 - x^2}}$$

(a) $(-\infty, \infty)$

(b) $x \neq 0$

(c) $0 < x < 6$

(d) $x \neq \pm 36$

(e) $x \neq \pm 6$

1.6 Given $f(x) = x^2 + 6x$. Find the intervals on which f is increasing or decreasing. (1)

(a) Increasing on $(-\infty, -3)$; decreasing on $(-3, \infty)$

(b) Increasing on $(-6, \infty)$; decreasing on $(-\infty, -6)$

(c) Increasing on $(-3, \infty)$; decreasing on $(-\infty, -3)$

(d) Increasing on $(-\infty, -6)$; decreasing on $(-6, \infty)$

(e) No increasing or decreasing interval

1.7 Determine where the graph of the function $f(x) = x^3 + 30x$ is concave upward (CU) and where it is concave downward (CD). (2)

(a) CU on $(0, \infty)$, CD on $(-\infty, 0)$

(b) CU on $(-\sqrt{10}, \sqrt{10})$, CD on $(-\infty, -\sqrt{10})$ and $(\sqrt{10}, \infty)$

(c) CU on $(\sqrt{10}, \infty)$, CD on $(-\infty, -\sqrt{10})$

(d) CU on $(-\sqrt{10}, \infty)$, CD on $(-\infty, -\sqrt{10})$

(e) CU on $(0, \sqrt{10})$, CD on $(-\sqrt{10}, 0)$

1.8 Choose an equation from the following that expresses the fact that a function f is continuous at the number 4. (1)

(a) $\lim_{x \rightarrow 4} f(x) = -\infty$

(b) $\lim_{x \rightarrow 4} f(x) = f(4)$

(c) $\lim_{x \rightarrow 4} f(x) = \infty$

(d) $\lim_{x \rightarrow 0} f(x) = f(4)$

(e) $\lim_{x \rightarrow 0} f(x) = 4$

1.9 Find the equation of the tangent line to the graph of the function at the indicated point (2)

$$f(x) = \frac{2}{x}, \quad (2, 1).$$

(a) $y = 2x + 2$

(b) $y = -2x + 2$

(c) $y = -\frac{1}{2}x + 2$

(d) $y = \frac{1}{2}x + 2$

(e) None of the above

1.10 Find $\frac{dy}{dx}$ by implicit differentiation (2)

$$e^{xy} - x^9 + y^9 = 3.$$

(a) $\frac{9x^8 - xe^{xy}}{ye^{xy} + 9y^8}$

(b) $\frac{9y^8 - ye^{xy}}{xe^{xy} + 9x^8}$

(c) $\frac{9y^8 - xe^{xy}}{ye^{xy} + 9x^8}$

(d) $\frac{9x^8 - ye^{xy}}{xe^{xy} + 9y^8}$

(e) None of the above

Question 2 [14 marks]

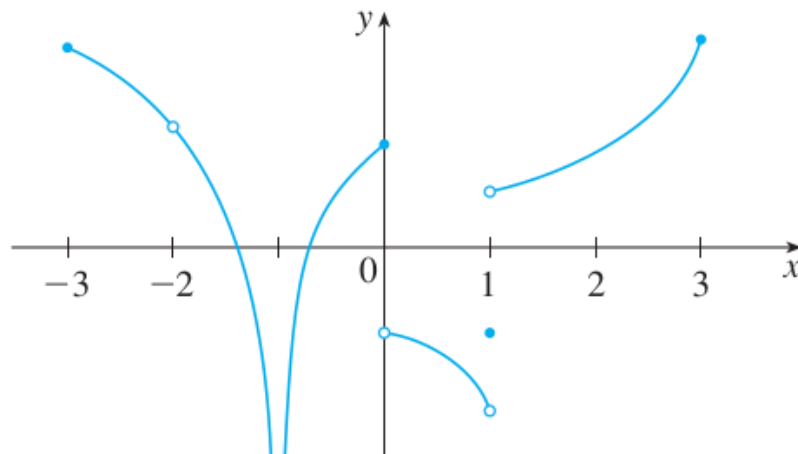
2.1 Sketch the graph of an example of a function f that satisfies all of the given conditions.

4

$$\lim_{x \rightarrow -1^-} f(x) = 0; \quad \lim_{x \rightarrow -1^+} f(x) = 1, \quad \lim_{x \rightarrow 2} f(x) = 3 \quad f(-1) = 2, \quad f(2) = 1$$

2.2 From the graph of g , state the numbers at which g is discontinuous and explain why.

4



2.3 Evaluate the limit or show that it does not exist

3

$$\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$$

2.4 For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

3

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

Question 3 [12 marks]

3.1 Find the derivative of this function using the definition of derivative.

4

$$f(x) = \frac{x^2 - 1}{2x - 3}$$

State the domain of $f(x)$ and $f'(x)$

1

3.2 Use implicit differentiation to find an equation of the tangent line to this curve at the given point

$$2(x^2 + y^2)^2 = 25(x^2 - y^2), \quad (3, 1).$$

4

3.3 Find the antiderivative f of f'' that satisfies the given condition.

3

$$f'' = -2 + 12x - 12x^2, \quad f(0) = 4, \quad f'(0) = 12$$

Question 4 [18 marks]

4.1 A manufacturer has been selling 1000 flat-screen TVs a week at R450 each. A market survey indicates that for each R10 rebate offered to the buyer, the number of TVs sold will increase by 100 per week.

(a) Find the demand function. 2

(b) How large a rebate should the company offer the buyer in order to maximize its revenue? 3

(c) If its weekly cost function is $C(x) = 68000 + 150x$, how should the manufacturer set the size of the rebate in order to maximize its profit? 4

4.2 A runner sprints around a circular track of radius 100 m at a constant speed of 7 m/s. The runner's friend is standing at a distance 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 200 m?

(a) Make a sketch for the information given

2

(b) Find an equation for the distance between the friends and any associated equation

3

(c) Solve the equation(s) in (b) to get how fast the distance between the friends is changing when the distance between them is 200 m

4