Question 1 [20 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Give the polar coordinates of the point $(-1;1)$ where $r>0$ and $0\leq\theta\leq 2\pi$. (2)

(a) $\left(\sqrt{2};-\frac{\pi}{4}\right)$

(b) $\left(\sqrt{2};\frac{\pi}{2}\right)$

(c) $\left(\sqrt{2};\frac{3\pi}{4}\right)$

(d) $\left(\sqrt{2};-\frac{\pi}{2}\right)$

(e) None of the above.

1.2 Which of the following equations are linear?

(i) $x+5\sqrt{y}-2z=1$ (ii) $x+3y+xz=2$ (iii) $x=-7y+3z$ (iv) $e^x-z=4$
(v) $\pi x+\sqrt{2}y+\frac{1}{3}z=7^{1/3}$. (2)

(a) (i), (iii) and (v)

(b) (ii) and (iii)

(c) (iii) only

(d) (iii) and (iv)

(e) None of the above.

1.3 The arc length of $y = 2e^x \sin x$ on the interval $\left[0, \frac{3\pi}{2}\right]$ equals? (2)

(a) $\displaystyle\int_0^{\frac{3\pi}{2}} \sqrt{1 + 4e^x(1 + \sin 2x)}\, dx$

(b) $\displaystyle\int_0^{\frac{3\pi}{2}} \sqrt{1 + 2e^x(\sin x + \cos x)}\, dx$

(c) $\displaystyle\int_0^{\frac{3\pi}{2}} \sqrt{1 + 4e^{2x}(1 + \sin 2x)}\, dx$

(d) $\displaystyle\int_0^{\frac{3\pi}{2}} \sqrt{1 + 4e^{2x}(1 - \sin x)^2}\, dx$

(e) None of the above.

1.4 The correct trigonometric substitution for $\displaystyle\int \frac{x^3}{\sqrt{4 + x^2}}\, dx$ leads to: (2)

(a) $\displaystyle\int 4 \tan^3 \theta \, d\theta$

(b) $\displaystyle\int 8 \tan^3 \theta \, d\theta$

(c) $\displaystyle\int 4 \tan^3 \theta \sec \theta \, d\theta$

(d) $\displaystyle\int 8 \tan^3 \theta \sec \theta \, d\theta$

(e) None of the above.

1.5 Which of the integrals below gives the area of the surface obtained when the region under the curve $y = \frac{2}{x}$, $x \in [1, 5]$ is rotated about the $x-$axis? (2)

(a) $\displaystyle 4\pi \int_1^5 x^{-3}\sqrt{x^4 + 4}\, dx$

(b) $\displaystyle 2\pi \int_1^5 \frac{\sqrt{x^2 - 4}}{x^3}\, dx$

(c) $\displaystyle 4\pi \int_1^5 \frac{\sqrt{x^2 + 4}}{x^5}\, dx$

(d) $\displaystyle 2\pi \int_1^5 x\sqrt{1 + \frac{4}{x^2}}\, dx$

(e) None of the above.

1.6 The correct partial fraction decomposition for $\dfrac{x+1}{(x^2-4)(x^2+2)}$ is: (2)

(a) $\dfrac{A}{x^2-4}+\dfrac{Bx+C}{x^2+2}$

(b) $\dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{C}{x^2+2}$

(c) $\dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+2}$

(d) $\dfrac{Ax}{x^2-4}+\dfrac{Bx+C}{x^2+2}$

(e) None of the above.

1.7 The point on the curve $y=\sqrt{x}$ that is closest to the point (2,0) is: (2)

(a) $\left(\sqrt{\dfrac{3}{2}},\dfrac{3}{2}\right)$

(b) $\left(\dfrac{3}{2},\sqrt{\dfrac{3}{2}}\right)$

(c) $\left(\sqrt{\dfrac{5}{2}},\dfrac{5}{2}\right)$

(d) $\left(\sqrt{\dfrac{1}{2}},\dfrac{1}{2}\right)$

(e) None of the above.

1.8 Suppose $y=\sqrt{3x+1}$ where x and y are functions of t. If $\dfrac{dx}{dt}=10$, find $\dfrac{dy}{dt}$ when $x=8$. (2)

(a) 1

(b) 5

(c) 3

(d) 10

(e) None of the above.

1.9 $\displaystyle\sum_{k=0}^{n} \binom{n}{k} 9^k$ is equal to: (2)

(a) 10^n

(b) 9^n

(c) 9^k

(d) 10^k

(e) None of the above.

1.10 If two equations have no values to satisfy both equations, then this is called: (2)

(a) Consistent system
(b) Inconsistent system
(c) Solution system
(d) Constant system
(e) None of the above.

Question 2 [7 marks]

(a) **Set up, but do not evaluate**, an integral for the length of the curve: (2)
$y = \ln(g(x) - 1)$, $g(x) > 1$, $x \in [4, 9]$.

(b) The curve $y = \sqrt{1 + e^x}$, $0 \leq x \leq 1$, is rotated about the $x-$axis. Find the area of the resulting surface. (5)

Question 3 [3 marks]

Find all the points on the parametric curve shown below with slope equal to $\frac{1}{\sqrt{3}}$.

$$x = a \cos t, \quad y = a \sin t.$$

Question 4 [6 marks]

Evaluate each of the following integrals if they exist.

(a) $\displaystyle\int_1^e x^2 \ln(4x)\,dx.$ (3)

(b) $\displaystyle\int_7^5 \frac{dx}{\sqrt{7-x}}.$ (3)

Question 5 [7 marks]

(a) Solve the differential equation shown below: (3)

$$y' = \frac{1+2y^2}{(x^2-1)xy}, \quad x > 1.$$

(b) Solve the differential equation shown below: (2)

$$\frac{dy}{dx} + f'(x)y = f'(x).$$

(c) Suppose $f'(x) = \sec^2 x$ and that $y\left(\frac{\pi}{4}\right) = 0$. Find a solution to the differential equation from **Question 5b**. (2)

Question 6 [7 marks]

(a) Sketch the parametric curve below and indicate with an arrow the direction in which the curve is traced as t increases: (3)

$$x(t) = \frac{2}{t},\ y(t) = -\frac{4}{t^2},\ 1 \leq t \leq 4.$$

(b) **Set up, but do not evaluate**, an integral for the arc length of the curve: (2)

$$x(t) = \frac{3}{t}, \quad y(t) = \frac{9}{t^2}, \quad 1 \leq t \leq 4.$$

(c) **Set up, but do not evaluate**, an integral for the surface area obtained by rotating the curve shown below about the $y-$axis. Simplify the integrand as far as possible. (2)

$$x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le 5.$$

Question 7 [10 marks]

(a) Give a system of linear equations corresponding to the given augmented matrix: (2)

$$\begin{bmatrix} 3 & 0 & 1 & 5 \\ 0 & -1 & 4 & 2 \\ 5 & -2 & 1 & 7 \end{bmatrix}$$

(b) Find the augmented matrix for the given system of linear equations: (2)

$$x_2 - x_6 = 7$$
$$x_3 = -1$$

(c) Solve the system below using **Gauss–Jordan Elimination**. (6)

$$2x + 3y = -3 - 2z$$
$$y = -x - z$$
$$-x + 2y + 1 = 3z$$

Question 8 [5 marks]

Use Binomial Theorem to find the **coefficient** of x^3 in the expansion of $(x-3)(2x+1)^6$.

Question 9 [5 marks]

Consider the matrices shown below:

$$B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

Find B^{-1} and the value of K such that:

$$B^{-1}C = \frac{4}{5K} \begin{bmatrix} 10 & 18 & 18 \\ 24 & 8 & 40 \end{bmatrix}$$