Question 1 [15]

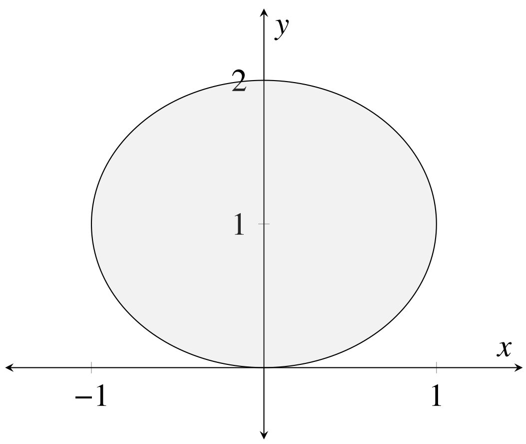
Choose the correct option for the multiple choice questions below and write your answer in the table provided.

Question	Answer
1.1	
1.2	
1.3	
1.4	
1.5	
1.6	
1.7	
1.8	
1.9	

(1.1) The maximum directional derivative of $f(x, y) = ye^{xy}$ at $(0, 2)$ is: (2)

(a) 4 (b) 5 (c) $\sqrt{17}$ (d) $3\sqrt{2}$ (e) 3

(1.2) An iterated integral which represents the area of the region below is given by: (1)



(a) $\int_0^{2\pi}\int_0^{2\sin\theta} r\,drd\theta$ (b) $\int_{\pi/4}^{3\pi/4}\int_0^{2\sin\theta} r\,drd\theta$ (c) $\int_0^{\pi}\int_0^{2\cos\theta} r\,drd\theta$ (d) $\int_0^{\pi}\int_0^{2\sin\theta} r\,drd\theta$

(1.3) Let f be a scalar field and let $\mathbf{F}$ be a vector field. Which of the expressions below are meaningful and results in a vector field: (1)

(i) $\operatorname{curl} f$

(ii) $\nabla f \times \operatorname{div}\mathbf{F}$

(iii) $\operatorname{div}(\operatorname{curl}(\nabla f))$

(iv) $\operatorname{curl}(\operatorname{curl}\mathbf{F})$

(v) $\operatorname{curl}\mathbf{F}\cdot\operatorname{curl}\mathbf{F}$

(a) i, v (b) iii, v (c) iv (d) iv, v (e) None of these choices

(1.4) Evaluate the line integral given by

$$\oint_C y^3\,dx - x^3\,dy,$$

where C is the circle $x^2 + y^2 = 4$. (2)

(a) -12π (b) -24π (c) 24π (d) 18π (e) -18π

(1.5) If $\mathbf{F}(x, y, z) = z^2 y \sin x\mathbf{i} - z^2 \cos x\mathbf{j} - 2zy \cos x\mathbf{k}$, then curl $\mathbf{F}$ at $(0, 1, 2)$ is: (2)

(a) $\mathbf{0}$ (b) $-4\mathbf{i}$ (c) 4 (d) 0 (e) None of these choices

(1.6) Let $f(x, y) = e^{xy} + \ln\left(\frac{1}{x}\right)$ and let C be a semicircular path in the upper half of the plane from $(1, 0)$ to $(3, 0)$. Find $\int_C \nabla f \cdot d\mathbf{r}$. (2)

(a) $\ln 2 + 1$ (b) $\ln 2 - 1$ (c) $-\ln 3$ (d) $1 - \ln 3$ (e) None of these choices

(1.7) Name the quadratic surface represented by the equation $x^2 + 6z + 4x + y^2 - z^2 = 5$. (1)

(a) Ellipsoid (b) Cone

(c) Elliptic paraboloid (d) Hyperbolic paraboliod

(1.8) The domain and range of the following function

$$f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$$

is given by (2)

(a) $\{(x, y) \in \mathbb{R}^2 : \frac{x^2}{9} + \frac{y^2}{4} \leq 1\}; z \in [0, 6]$ (b)$\{(x, y) \in \mathbb{R}^2 : \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}; z \in [0, 6]$

(c) $\{(x, y) \in \mathbb{R}^2 : \frac{x^2}{4} + \frac{y^2}{9} \geq 1\}; z \in (0, 6)$ (d)$\{(x, y) \in \mathbb{R}^2 : \frac{x^2}{9} + \frac{y^2}{4} \geq 1\}; z \in (0, 6)$

(1.9) The limit of the following function

$$f(x, y) = \frac{xy + yz}{x^2 + y^2 + z^2}$$

as (x, y, z) approaches $(0, 0, 0)$ (2)

(a) equals 0 (b) equals $\frac{1}{2}$ (c) equals $-\frac{1}{2}$ (d) does not exist

Question 2 [5]

Determine whether the following statements are true or false. If true, give a short justification. If false, explain why or give a counter example.

(2.1) If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j}$ and $P_y = Q_x$ in an open region D, then $\mathbf{F}$ is conservative. (1)

(2.2)

$$f(x,y) = \begin{cases} \tan^{-1}\left(\frac{2y^2+x^2(\sin^2 y+1)}{2y^2+x^2}\right) & \text{if } (x,y) \neq (0,0) \\ \frac{\pi}{4} & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$. (1)

(2.3) A critical point is either a local minimum or a local maximum. (1)

(2.4) If $\int_0^{100} e^{-x^2}\,dx = I$, then $\int_0^{100}\int_0^{100} e^{-x^2}e^{-y^2}\,dy\,dx = I^2$. (1)

(2.5) For any vector fields $\mathbf{F}(x,y,z)$ and $\mathbf{G}(x,y,z)$, we have $\text{curl}(\mathbf{F}+\mathbf{G}) = \text{curl}\,\mathbf{F} + \text{curl}\,\mathbf{G}$. (1)

(2.6) For a function f to be continuous at (a,b) it is sufficient to check that f is defined at (a,b) and that $\lim_{(x,y)\to(a,b)} f(x,y)$ exists. (1)

Question 3 [6]

Instruction: Answer only ONE question from the following. Show full working of your solution.

- Find $\frac{\partial z}{\partial y}$ if $\sin(y) + x^2y - 2z^3y^3 = 3$.
- Use the Lagrange Multiplier method to maximize the function $x_1y_1 + x_2y_2 + x_3y_3$ subject to the constraints $x_1^2 + x_2^2 + x_3^2 = 1$ and $y_1^2 + y_2^2 + y_3^2 = 1$.

Question 4 [6]

Instruction: Answer only ONE question from the following. Show full working of your solution.

- Evaluate the integral $\iint_R \sqrt{x^2 + y^2}\, dA$, where R is the part of the unit disk in the first quadrant.
- Use a double integral and a convenient coordinate system to find the volume of the solid bounded by $z = e^{-x^2}$ and the planes $y = 0$, $y = x$, and $x = 1$.

Question 5 [6]

Express the volume of the solid that lies inside the cylinder $x^2 + y^2 = 2$, below the sphere $z^2 + y^2 + x^2 = 4$, and above the plane $z = 0$ as a triple integral in **one** of the specified coordinate system:

- Cylindrical coordinates.
- Spherical coordinates.

Question 6 [6]

Evaluate the integral by making an appropriate change of variables:

$$\iint_R \left(\frac{x-y}{x+y+2}\right)^2 \, dA,$$

where R is the square enclosed by the lines $x - y = -1$, $x + y = -1$, $x - y = 1$ and $x + y = 1$.

Question 7 [6]

Evaluate
$$\oint_C (3y - e^{\sin x})\,\mathrm{d}x + (7x + \sqrt{y^4 + 1})\,\mathrm{d}y,$$
Where C is the circle $x^2 + y^2 = 9$.

Question 8 [10]

Show that
(8.1)

$$\mathbf{F}(x,y) = \left\langle 4x^3y^2 - 2xy^3, 2x^4y - 3x^2y^2 + 4y^3 \right\rangle$$

is conservative and find the work done by $\mathbf{F}$ in moving a particle along the path C given by (6)

$$\mathbf{r}(t) = \langle t + \sin \pi t, 2t + \cos \pi t \rangle \ \ ; \ \ 0 \le t \le 1.$$

(8.2)

$$\operatorname{curl}(\mathbf{F}(x,y,z)) = \operatorname{curl}(\sqrt{y^4}z^3\mathbf{i} + \sqrt{4x^2y^2}z^3\mathbf{j} + \sqrt{9x^2y^4z^4}\mathbf{k}) = \mathbf{0}.$$

(4)