Question 1 [20 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The correct partial fraction decomposition for $\dfrac{x+1}{(x-1)^2(x^2+2)}$ is: (2)

(a) $\dfrac{A}{(x-1)^2} + \dfrac{Bx+C}{x^2+2}$

(b) $\dfrac{A}{(x-1)^2} + \dfrac{B}{x^2+2}$

(c) $\dfrac{Ax}{(x-1)^2} + \dfrac{Bx+C}{x^2+2}$

(d) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{Cx+D}{x^2+2}$

(e) None of the above.

1.2 The second term in the expansion of $(2x-y)^4$ is: (2)

(a) $32x^3y$
(b) $4x^3y$
(c) $-32x^3y$
(d) $-8x^2y^2$
(e) None of the above.

1.3 $\displaystyle\int_0^1 e^x \, dx$ equal: (2)

(a) e
(b) $\frac{1}{2}e^2 - \frac{1}{2}$
(c) $e-1$
(d) divergent
(e) None of the above.

1.4 Which equation is linear?

(i) $x + 5y - 2z = 1$ (ii) $x + 3y + xz = 2$ (iii) $x = -7y + 3z$ (iv) $e^x - z = 4$
(v) $\pi x + \sqrt{2}y + \frac{1}{3}z = 7^{1/3}$ (2)

(a) (i), (iii) and (v)

(b) (ii) and (iii)

(c) (iii)

(d) (iii) and (iv)

(e) None of the above.

1.5 The correct trigonometric substitution for $\displaystyle\int \frac{x^3}{\sqrt{9+x^2}}\,dx$ leads to: (2)

(a) $\displaystyle\int 9\tan^3\theta\,d\theta$

(b) $\displaystyle\int 27\tan^3\theta\,d\theta$

(c) $\displaystyle\int 9\tan^3\theta\sec\theta\,d\theta$

(d) $\displaystyle\int 27\tan^3\theta\sec\theta\,d\theta$

(e) None of the above.

1.6 The length of the edge of a cube is decreasing at a rate of 3 cm/s. We want to find the rate of change of the volume of the cube. If s is used to denote the edge of the cube, what two equations can be used to solve this problem? (2)

(a) $V = 6s^2$, $\dfrac{dV}{dt} = \dfrac{dV}{dt}\dfrac{ds}{dt}$.

(b) $V = 6s^2$, $\dfrac{dV}{dt} = \dfrac{dV}{ds}\dfrac{ds}{dt}$.

(c) $V = s^3$, $\dfrac{dV}{dt} = \dfrac{dV}{ds}\dfrac{ds}{dt}$.

(d) $V = s^3$, $\dfrac{ds}{dt} = \dfrac{dV}{ds}\dfrac{ds}{dt}$.

(e) None of the above.

1.7 Consider the surface of revolution formed by revolving the curve $y = \frac{1}{x}$, $1 \leq x \leq a$, about the $x - axis$. The surface area is: (2)

(a) $2\pi \displaystyle\int_1^a \frac{dx}{x}$

(b) $2\pi \displaystyle\int_1^a \frac{\sqrt{1+x^4}}{x^3}\, dx$

(c) $\displaystyle\int_1^a \frac{dx}{x^2}$

(d) $2\pi \displaystyle\int_1^a \sqrt{1+\frac{1}{x^4}}\, dx$

(e) None of the above.

1.8 Which integral below gives the arc length of the curve $y = \tan x$ on the interval $[0, \frac{\pi}{4}]$. (2)

(a) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1-\sec^4 x}\, dx$

(b) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1+\sec^4 x}\, dx$

(c) $\displaystyle\int_0^{1} \sqrt{\frac{\pi}{4}+\sec^4 x}\, dx$

(d) $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1+\tan^2 x}\, dx$

(e) None of the above.

1.9 If $y = \dfrac{x^3}{x^3+1}$ and $\dfrac{dy}{dx} = \dfrac{3x^2}{(x^3+1)^2}$ then the critical number(s) of the function y is: (2)

a) 0 and -1
b) -1 only
c) 0 and 1
d) 0 only
e) None of the above.

1.10 The area of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$ is: (2)

a) $\frac{16}{3}$

b) 12

c) $\frac{3}{16}$

d) 18

e) None of the above.

Question 2 [13 marks]

Consider the function $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$. Given that $f'(x) = \dfrac{-6x}{(x^2-4)^2}$ and $f''(x) = \dfrac{6(3x^2+4)}{(x^2-4)^3}$.

Complete the following for the function f.

(a) Domain. (1)

(b) Intercepts. (2)

(c) Asymptotes. (3)

(d) Interval of increasing and decreasing. (2)

(e) Local maximum and minimum values. (1)

(f) Concavity. (2)

(g) Sketch $f(x)$. (2)

<u>Question 3</u> [10 marks]

(a) **Set up, but do not evaluate**, an integral for the length of the curve below, simplifying as far as possible. (2)

$$y = \ln(1 - x^2), \quad 0 \leq x \leq \frac{1}{4}.$$

(b) The curve $y = \frac{1}{4}(x^2 - 2\ln x)$, $1 \leq x \leq 4$, is rotated about the $y-$axis. Find the area of the resulting surface. (4)

(c) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$ about the $y-$axis. Include a sketch of the region. (4)

Question 4 [6 marks]

Evaluate each of the following integrals if they exist.

(a) $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx.$ (3)

(b) $\displaystyle\int_3^2 \frac{dx}{\sqrt{3-x}}.$ (3)

Question 5 [7 marks]

(a) Sketch the parametric curve below and indicate with an arrow the direction in which the curve is traced as t increases. (3)

$$x(t) = \frac{2}{t},\ y(t) = \frac{4}{t^2},\ 1 \leq t \leq 4.$$

(b) **Set up, but do not evaluate**, an integral for the arc length of the curve below. (2)

$$x(t) = \frac{2}{t},\ y(t) = \frac{4}{t^2},\ 1 \leq t \leq 4.$$

(c) **Set up, but do not evaluate**, an integral for the surface area obtained by rotating the curve below about the $y-$axis. (2)

$$x = 3t^2,\ y = 2t^3,\ 0 \leq t \leq 5.$$

Question 6 [5 marks]

Expand the expression $\left(\sqrt{y}-\frac{1}{3}\right)^4$ using the Binomial Theorem. Simplify all the coefficients and exponents.

Question 7 [9 marks]

(a) Give a system of linear equations corresponding to the given augmented matrix: (2)

$$\begin{bmatrix} 3 & 0 & 0 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & -2 & -1 & 7 \end{bmatrix}$$

(b) Find the augmented matrix for the given system of linear equations: (1)

$$x_1 - x_5 = 7$$
$$x_2 = -1$$

(c) Solve the system using **Gauss–Jordan Elimination**. (6)

$$
\begin{aligned}
x + z &= 6 \\
-3y + z &= 7 \\
2x + y + 3z &= 15
\end{aligned}
$$