Exam information and instructions

Duration: 120 minutes **Marks:** 50
Assessor: Dr. G.J. Kemp
Moderator: Prof. I. Sinayskiy

Student number:

Surname & initials: _____

Question	Points	Score
1	15	
2	10	
3	10	
4	5	
5	10	
Total:	50	

Question 1 (15 marks)

This question concerns the Bloch sphere and the rotation matrices.

(a) Calculate the eigenvectors of the three Pauli matrices, X, Y, Z. Draw the Bloch sphere and indicate the positions of each eigenvector on the Bloch sphere. (6)

(b) Rotate the vector (6)

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

by an angle $\pi/2$ about the z-axis. Where is this new vector on the Bloch sphere?

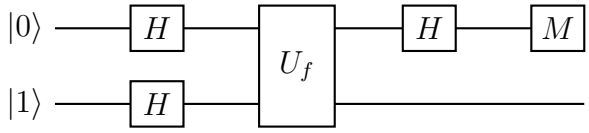
(c) What is the net effect of the product $R_x(\pi/2)R_x(2\pi/3)$? (3)

Question 2 (10 marks)

The Deutsch Algorithm, as shown below for two qubits, can be used to determine global properties of a function, f. For example, it can determine if the function $f : [0, 1] \longrightarrow [0, 1]$ is balanced or constant.

(a) Calculate the final state of the system if the function f is balanced, just before the measurement, M? (6)

(b) Explain how the measurement M on the top qubit will determine whether f is balanced or constant. (4)



Question 3 (10 marks)

This question concerns the Schmidt decomposition.

(a) What is the Schmidt decomposition for the state (5)

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right)?$$

(b) What is the Schmidt decomposition for the state (5)

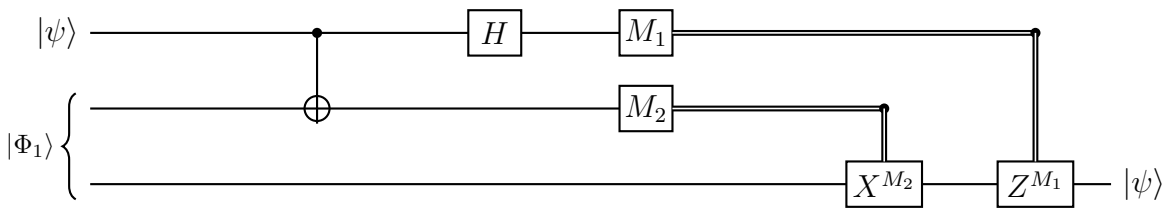
$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + i\,|11\rangle\right)?$$

Question 4 (5 marks)

Draw the quantum circuit necessary to produce the Bell state $|\Phi_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$.
How should this circuit be modified to then produce the Bell state $|\Phi_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$?
Draw this circuit.

Question 5 (10 marks)

The following circuit implements the quantum teleportation algorithm.



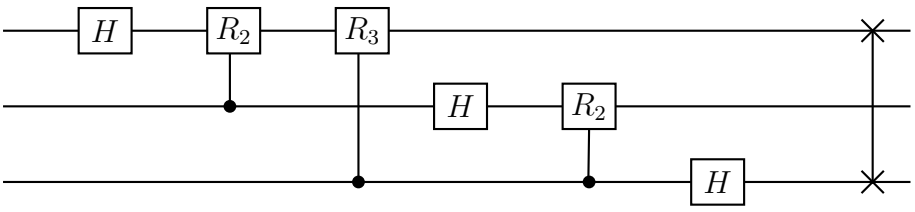
The top two registers belong to Alice and the bottom one belongs to Bob. Alice and Bob share the entangled state $|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice wants to transmit the qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob using the above circuit.

(a) Calculate the overall state just before Alice performs the measurements M_1 and M_2. (7)

(b) Say Alice performs a measurement on her two qubits and obtains the result $M_1 = 1, M_2 = 0$. What is the probability of obtaining this result? Describe the transformation that Bob must now apply to his qubit to obtain the desired state ψ. (3)

Bonus Question

For an extra 5 marks, consider the circuit below,



and calculate the output state for the input state $|j_1 j_2 j_3\rangle = |100\rangle$.

 END OF QUESTION PAPER