ASMAVIA NOVEMBER EXAM 2021

Q1

1.1 D

1.2 C

1.3 C

1.4 A

1.5 C

1.6 B

1.7 A

1.8 B

1.9 B

1.10 A

Q2

$$
\begin{aligned}
LHS &= \tan^2 x - \sin^2 x \\
&= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x \\
&= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} \\
&= \frac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x} \\
&= \frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x \\
&= \tan^2 x \sin^2 x \\
&= RHS
\end{aligned}
$$

Q3

$$x + y + z = 6$$
$$2y + 5z = -4$$
$$2x + 5y - z = 27$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 2 & 5 & -1 & 27 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \qquad A_x = \begin{bmatrix} 6 & 1 & 1 \\ -4 & 2 & 5 \\ 27 & 5 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 6 & 1 \\ 0 & -4 & 5 \\ 2 & 27 & -1 \end{bmatrix} \qquad A_z = \begin{bmatrix} 1 & 1 & 6 \\ 0 & 2 & -4 \\ 2 & 5 & 27 \end{bmatrix}$$

$$|A| = -0\begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} - 5\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$$
$$= 0 + 2(-3) - 5(3)$$
$$= -21$$

$$|A_x| = 6\begin{vmatrix} 2 & 5 \\ 5 & -1 \end{vmatrix} - \begin{vmatrix} -4 & 5 \\ 27 & -1 \end{vmatrix} + \begin{vmatrix} -4 & 2 \\ 27 & 5 \end{vmatrix}$$
$$= 6(-27) - (-131) + (-74)$$
$$= -105$$

$$|A_y| = \begin{vmatrix} -4 & 5 \\ 27 & -1 \end{vmatrix} - 0\begin{vmatrix} 6 & 1 \\ 27 & -1 \end{vmatrix} + 2\begin{vmatrix} 6 & 1 \\ -4 & 5 \end{vmatrix}$$
$$= -131 - 0 + 2(34)$$
$$= -63$$

$$|A_z| = \begin{vmatrix} 2 & -4 \\ 5 & 27 \end{vmatrix} - 0\begin{vmatrix} 1 & 6 \\ 5 & 27 \end{vmatrix} + 2\begin{vmatrix} 1 & 6 \\ 2 & -4 \end{vmatrix}$$
$$= 74 - 0 + 2(-16) = 42$$

$$x = \frac{|A_x|}{|A|} = 5, \quad y = \frac{|A_y|}{|A|} = 3, \quad z = \frac{|A_z|}{|A|} = -2$$

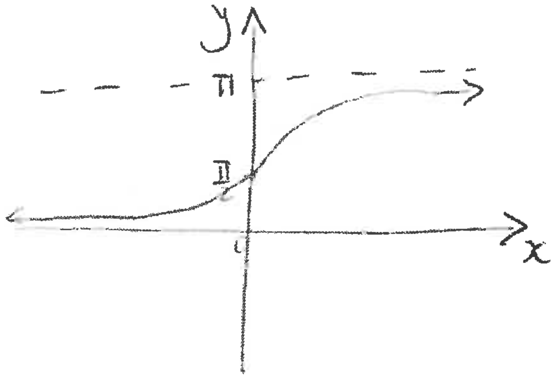
$$\therefore (x, y, z) = (5, 3, -2)$$

Q4 (a) $f(x) = \frac{1}{\tan x}$

$f(-x) = \frac{1}{\tan(-x)} = -\frac{1}{\tan x} = -f(x)$

$\therefore$ f is odd

(b)



Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

Q5 $2^{\log(x+1)} = \sin\frac{\pi}{2}$

$2^{\log(x+1)} = 1$

$2^{\log(x+1)} = 2^0$

$\log(x+1) = 0$

$x+1 = 1$

$x = 0$

Q6 (a) $f(x) = \frac{\sqrt{9x^6 - x}}{x^3+1}$

$x^3+1 \neq 0$

$\Rightarrow x^3 \neq -1$

$\Rightarrow x \neq -1$

$\therefore x = -1$ is a VA.

Q6 (b)

$$\lim_{x \to \infty} \frac{\sqrt{9x^6 - x}}{x^3+1} = \lim_{x \to \infty} \frac{\left(\frac{\sqrt{9x^6 - x}}{x^3}\right)}{1 + \frac{1}{x^3}}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{\sqrt{9x^6 - x}}{\sqrt{x^6}}\right)}{1 + \frac{1}{x^3}} \quad ; \ |x^3| = \sqrt{x^6} \text{ for } x^3 > 0$$

$$= \lim_{x \to \infty} \frac{\sqrt{9 - \frac{1}{x^5}}}{1 + \frac{1}{x^3}}$$

$$= 3$$

$$\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3+1} = -\lim_{x \to -\infty} \frac{\frac{\sqrt{9x^6 - x}}{\sqrt{x^6}}}{1 + \frac{1}{x^3}} \quad |x^3| = -\sqrt{x^6} \text{ for } x^3 < 0$$

$$= -\lim_{x \to -\infty} \frac{\sqrt{9 - \frac{1}{x^5}}}{1 + \frac{1}{x^3}}$$

$$= -3$$

$\therefore$ $y = \pm 3$ are HAs.

Q7 $f(1) = 3(1^2) = 3$

$\lim_{x \to 1^-} (3x^2) = 3$ and $\lim_{x \to 1^+} (4-x) = 3$

$\Rightarrow \lim_{x \to 1} f(x) = 3$

Since $\lim_{x \to 1} f(x) = 3 = f(1)$ $\therefore$ f is continuous at $x=1$.

Q8 $-1 \leq \cos x \leq 1$

$1 \geq -\cos x \geq -1$

$-1 \leq -\cos x \leq 1$

$1 \leq 2-\cos x \leq 3$

$x \to \infty \Rightarrow x+3 > 0$

$\Rightarrow \frac{1}{x+3} \leq \frac{2-\cos x}{x+3} \leq \frac{3}{x+3}$

$\lim_{x \to \infty} \frac{1}{x+3} = 0 = \lim_{x \to \infty} \frac{3}{x+3}$

$\therefore \lim_{x \to \infty} \frac{2-\cos x}{x+3} = 0$. By Squeeze Theorem

Q 9(a)

$$y = (\sqrt{x})^{\sqrt{x}}$$

$$\ln y = \ln (\sqrt{x})^{\sqrt{x}}$$

$$\ln y = \sqrt{x} \ln \sqrt{x}$$

$$\ln y = \frac{\sqrt{x}}{2} \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sqrt{x}}{2} \cdot \frac{1}{x} + \ln x \cdot \frac{1}{4\sqrt{x}}$$

$$\frac{dy}{dx} = (\sqrt{x})^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} + \frac{\ln x}{4\sqrt{x}} \right]$$

$$= \frac{1}{2}(\sqrt{x})^{\sqrt{x}-1} \left[1 + \frac{1}{2}\ln x \right]$$

(b)

$$x^4 + 4xy + y^4 = 6$$

$$\frac{d}{dx}(x^4 + 4xy + y^4) = \frac{d}{dx}(6)$$

$$4x^3 + 4x\frac{dy}{dx} + 4y + 4y^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^3 - y}{x + y^3}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = \frac{-2}{2} = -1$$

Equation: $y - y_1 = m(x - x_1)$

$$y - 1 = -1(x-1)$$

$$y = -x + 2.$$

Q10 $\quad (x^2 - \frac{3}{y})^5 = \sum_{r=0}^{5} (x^2)^{5-r} (-\frac{3}{y})^r \binom{5}{r}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$= \binom{5}{0}(x^2)^5(-\tfrac{3}{y})^0 + \binom{5}{1}(x^2)^4(-\tfrac{3}{y})^1 + \binom{5}{2}(x^2)^3(-\tfrac{3}{y})^2$$

$$+ \binom{5}{3}(x^2)^2(-\tfrac{3}{y})^3 + \binom{5}{4}(x^2)^1(-\tfrac{3}{y})^4 + \binom{5}{4}(x^2)^0(-\tfrac{3}{y})^5$$

$$= 1 \cdot x^{10} + 5 \cdot x^8 (-\tfrac{3}{y}) + 10x^6(\tfrac{9}{y^2}) + 10x^4(-\tfrac{27}{y^3})$$

$$+ 5x^2(\tfrac{81}{y^4}) + (-\tfrac{243}{y^5})$$

$$= x^{10} - 15\frac{x^8}{y} + 90\frac{x^6}{y^2} - 270\frac{x^4}{y^3} + 405\frac{x^2}{y^4}$$

$$- \frac{243}{y^5}$$

Q11 $\quad \int x^3 \sqrt{x^2+K}\, dx$

Let $u = x^2 + K \Rightarrow x^2 = u - K$

$\frac{du}{dx} = 2x$

$$\int x^3 \sqrt{x^2+K}\, dx = \int x^3 u^{1/2} \cdot \frac{du}{2x}$$

$$= \int x^2 u^{1/2}\, du$$

$$= \int (u-K) u^{1/2} du$$

$$= \int (u^{3/2} - K u^{1/2}) du$$

$$= \frac{u^{5/2}}{5} - K\frac{u^{3/2}}{3} + C$$

$$= \frac{(x^2+K)^{5/2}}{5} - \frac{K(x^2+K)^{3/2}}{3} + C$$