UNIVERSITY OF JOHANNESBURG

ENGINEERING CALCULUS 2B1

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS
UNIVERSITY OF JOHANNESBURG

ASSESSMENT: FINAL EXAMINATION - 10/2021	MODULE CODE: MAT0CB2/MATECB2
TOTAL POINTS: 40	ASSESSORS: DR. M. SIAS
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Instructions

- Before writing the test, make sure you have read the following honesty declaration and agree with it.
- Answer all questions. Show all the steps of your work.

Honesty Declaration

By writing this test, you confirm that you have not

- committed academic misconduct in any form;
- committed plagiarism;
- helped or attempted to help another student in preparing their submission for the assessment;
- misrepresented someone else's work as your own;
- obtained help or attempted to obtain help from another person;
- obtained help or attempted to obtain help from any source of information, except for explicitly provided lecture notes by the module assessor;
- made use of solutions or answers produced on websites.

QUESTION 1. [$2\times 5=10$]

Check whether the following statements are true or false. Write T if it is true or write F if it is false. Do not include explanations.

(i) If $f(x,y)=\sqrt{4-x^2-4y^2}$, then $f_x(1,0)=\dfrac{1}{\sqrt{2}}$

[F; $\dfrac{-1}{\sqrt{3}}$]

(ii) The gradient of the function $f(x,y)=x^2\ln y$ at $(3,1)$ is $9j$.
[T]

(iii) If $R=[1,2]\times[0,\pi]$, then $\iint_R y\sin(xy)\,dA=1$.
[F; 0]

(iv) $\int\limits_0^1\int\limits_0^{s^2}\cos(s^3)\,dt\,ds=\dfrac{1}{2}\sin 3$.
[F; $\dfrac{1}{3}\sin 1$.]

(v) $F(x,y)=(xy+y^2)\mathbf{i}+(x^2+2xy)\mathbf{j}$ is a conservative vector field.
[F].

QUESTION 2. [5]

Find the extreme values of f subject to both constraints.

$$f(x,y,z)=x+y+z,\quad x^2+z^2=2,\quad x+y=1.$$

QUESTION 3. [4]

Use polar coordinates to combine the following sum of integrals into a single double integral and then evaluate the integral.

$$\int_0^1\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}}\sin(x^2+y^2)\,dy\,dx+\int_0^{\sqrt{3}}\int_1^{\sqrt{4-y^2}}\sin(x^2+y^2)\,dx\,dy.$$

QUESTION 4. [5]

Consider the volume represented by the following triple integral.

$$V=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}dz\,dy\,dx-\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}}dz\,dy\,dx.$$

(i) Explain, in words, the represented volume. (1)

(ii) Rewrite the *first term only* in the order $dx\,dz\,dy$. (2)

(iii) Rewrite V in *spherical coordinates* using only one triple integral. (2)

Question 5. [5]

Evaluate the integral by making an appropriate change of variables:

$$\iint_R \left(\frac{x-y}{x+y+2}\right)^2 dA,$$

where R is the square enclosed by the lines $x-y=-1$, $x+y=-1$, $x-y=1$ and $x+y=1$.

Question 6. [6]

Show that

$$\mathbf{F}(x,y)=\left\langle 4x^3y^2-2xy^3, 2x^4y-3x^2y^2+4y^3\right\rangle$$

is conservative and find the work done by $\mathbf{F}$ in moving a particle along the path C given by

$$\mathbf{r}(t)=\langle t+\sin\pi t, 2t+\cos\pi t\rangle\ ;\ 0\leqslant t\leqslant 1.$$

Question 7. [5]

Let $\mathbf{r}=\langle x,y,z\rangle$ and let $r=|\mathbf{r}|$. Determine the value(s) of p such that the vector field $\mathbf{F}=\dfrac{\mathbf{r}}{r^p}$ has divergence zero.