UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	MATECB2 / MATOCB2 MULTIVARIABLE AND VECTOR CALCULUS FOR ENGINEERS
CAMPUS	APK
ASSESSMENT	EXAMINATION

DATE 18/01/2021	TIME 08:00
ASSESSOR(S)	MR M SIAS DR C ROBINSON
INTERNAL MODERATOR	DR A GOSWAMI
DURATION 120 MINUTES	MARKS 40

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 8 PAGES

INSTRUCTIONS:

1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [5 marks]

You have decided to build a fish tank to keep piranhas. The rectangular base and four sides will be made of shatterproof glass costing R800 per square metre. The top will remain open. Use the method of Lagrange multipliers to calculate the maximum cost of a tank of volume 0.5 m^3. (Note: 0.5 m^3 is just over 130 gallons, the minimum recommended size for keeping five piranhas.)

Question 2 [5 marks]

Consider the following integral:

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx.$$

(a) Sketch the region of integration. (1)

(b) Reverse the order of integration. (2)

(c) Evaluate the integral. (2)

Question 3 [5 marks]

Consider the curve $x^2 + y^2 = 2x$ in the xy-plane.

(a) Convert the curve to polar coordinates. (1)

(b) Express the enclosed area as a double integral (in polar **or** rectangular coordinates). (2)

(c) Using a method of your choice, calculate this area. (2)

Question 4 [5 marks]

Consider a solid Q that is bounded above above by the graph $4x^2 + 4y^2 + z^2 = 16$ and below by the graph $z = \sqrt{4x^2 + 4y^2}$. Sketch Q and set up a triple integral in spherical coordinates to find the volume of Q (do not evaluate the iterated integral).

Question 5 [6 marks]

Use the transformation $x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$ to evaluate

$$\iint_R (4x + 8y)\, dA,$$

where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$ and $(1, 5)$.

Question 6 [6 marks]

Given that

$$\mathbf{F}(x, y, z) = yze^{xz}\,\mathbf{i} + e^{xz}\,\mathbf{j} + xye^{xz}\,\mathbf{k}.$$

(a) Find a function f such that $\mathbf{F} = \nabla f$. (3)

(b) Use (a) to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C given by (3)

$$\mathbf{r}(t) = \langle t^2 + 1, t^2 - 1, t^2 - 2t \rangle \qquad 0 \leq t \leq 2.$$

Question 7 [5 marks]

Use Green's Theorem to evaluate the line integral along the given positively oriented curve:

$$\int_C (y + e^{\sqrt{x}})\,dx + (2x + \cos y^2)\,dy,$$

where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Question 8 [3 marks]

If C is a piecewise-smooth simple closed curve and f and g are differentiable functions, show that

$$\int_C f(x)\,dx + g(y)\,dy = 0.$$