FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

BACHELOR OF ENGINEERING TECHNOLOGY
(Chemical, Civil, Construction, Electrical, Industrial, Mechanical, Metallurgy)

Engineering Mathematics 1B - MATE1B1

Assessment: Supplementary Examination

Total Marks: 65

Duration: 3 hours

Date: 29 November 2021

Time: 11:30

Assessors : Dr P. Dlamini, Mr E. Morapeli, Dr S. Simelane, Mr E. Singh

Moderator : Dr R. Durandt

Instructions :

- Answer all questions.
- Show every detail of your working. Failure to do so will result in significant loss of marks. Note that you will be awarded zero if you only give the final answer.
- Simplify your answers fully.

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1. Find $\frac{dy}{dx}$, given that

$$y = \sqrt{\frac{\sin x}{xe^x}}$$

[4]

2. A curve C is given by the parametric equations

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t for the curve C [5]

3. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of x and y if $\sin^2(7y) - \cos^2(7y) = 14x$. [5]

4. Verify that $f(x,t) = e^{-rt}\sin(x+ct)$ satisfies the driven transport equation $f_t(x,t) = cf_x(x,t) - rf(x,t)$ [4]

5. The combined resistance R of two wires in parallel, having resistances R_1 and R_2 respectively, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Suppose R is $1.5\mathrm{ohms}$ and needs to be reduced to $0.7\mathrm{ohms}$. By how much should R_1 change to achieve this, if R_2 is changed from $2\mathrm{ohms}$ to $0.5\mathrm{ohms}$? Use calculus to find the change in R_1. [7]

6. Evaluate the following integral:

$$\int 2^x 4^x \sin(x) dx$$

[5]

7. Evaluate the following integral:

$$\int \sin^2(x)(1-\sin^2(x))^{10}\sin(x) dx$$

[3]

8. Evaluate the following integral:

$$\int \frac{x+7}{x(x-1)(x-2)(x-3)} dx$$

[5]

9. Find the area bounded by the graphs $y = x+6$, $y = x^3$ and $y = -\frac{1}{2}x$. First sketch the curves and indicate the required area, clearly indicating all intercepts and/or points of intersection. [5]

10. Use the shell method to compute the volume of the solid obtained by rotating the region enclosed by the graphs of the functions $y = x^2$, $y = 8 - x^2$ and to the right of $x = 1.5$ about the y-axis. [5]

11. Let $\alpha = (5, -2, 1); \beta = (1, 6, 3)$ and $k = -4$. Show that

    (a) $k(\alpha \cdot \beta) = (k\alpha) \cdot \beta$ [2]

    (b) $\|k\alpha\| = |k|\|\alpha\|$ [2]

12. Suppose A, B, and C are matrices with orders $(2 \times 3), (3 \times 3)$ and (2×2). Determine if the following matrix expression is defined. If it is defined, give its order:

$$BA + C$$

[2]

13. If $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 4 & 0 \\ 17 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & -1 \end{bmatrix}$

    Compute the element of AB in the **second row** and **first column**. [1]

14. Given the system

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

    (a) Find the matrices A and B such that $AX = B$ [1]

    (b) Use GAUSSIAN elimination to find A^{-1} [3]

    (c) Solve for x, y, z using this inverse [1]

15. Find the eigenvalues and eigenvector of the **smallest** eigenvalue for

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

[5]