University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	**MAT01B2/MAT2B10** MULTIVARIABLE AND VECTOR CALCULUS
CAMPUS	**APK**
EXAMINATION	**9 NOVEMBER 2021**

Availability	9 NOVEMBER 2021 16:30 to 19:00
Assessors	Dr C Robinson
Moderator	Dr M Sias
Total	50 Marks

Number of Pages: 2+2 Pages

Instructions:

1. Read all the questions carefully (from start to finish) before attempting to solve them.
2. Answer all questions.
3. Show all calculations and motivate all answers.
4. There are several different versions of the exam. You have only been allocated one of the versions. You must download the questions yourself and are not allowed to obtain them by any other means than via Blackboard. You are not allowed to share your downloaded copy of the questions with any other student. You MUST answer and submit the version allocated to you.
5. You will be allowed unlimited opportunities to upload a file. However, the LAST attempt will be marked. Please make sure that you select the correct file.
6. Please submit a single pdf file in which all pages have the same, upright, orientation.
7. Write your student number and surname on the top of each page of your submission.
8. Absolutely no teamwork or external assistance is allowed–if there is any indication that your work resembles another student's work, or is not your own work, the case will be sent for analysis and a disciplinary hearing and possible expulsion may result. You are not allowed to make use of online-solvers/calculators or pay tutors/other students (internal or external) to complete the exam on your behalf.
9. Please allow some time for uploading and submitting your solutions. That is, please do not leave it to the last minute to submit in case you find that you have connectivity issues.
10. Good luck!

Question 1 [2]

Suppose g is an integrable unary function and

$$f(x, y) = \int_y^x g(t)\, dt.$$

Determine $f_y(x, y)$.

Question 2 [4]

Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2y + xy^2}$ or show that it does not exist.

Question 3 [6]

Find the points on the cone $z^2 = x^2 + 2y^2$ that are closest to the point $(1, 2, 0)$. (Hint: You can minimize the square of the distance to make the calculations easier.)

Question 4 [4]

Let the polar coordinate system be changed (for this problem only) by the introduction of a factor 2 into the y coordinate, $x = r\cos\theta$, $y = 2r\sin\theta$, so that the curves $r = c$ are no longer circles, but ellipses, and call these coordinates (r, θ) elliptic polar coordinates.

(a) Compute the Jacobian $\dfrac{\partial (x, y)}{\partial (r, \theta)}$. (2)

(b) Use part (a) to set up a double integral in elliptic polar coordinates that represents the area of the ellipse with major radius 1 and minor radius 2. Evaluate the integral. (2)

Question 5 [9]

Let E be the solid that lies above the cone $z = \sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$ and below the sphere $x^2+y^2+z^2 = \dfrac{16}{3}$.

(a) Sketch the solid E. (1)

(b) Using symmetry, **set up** a triple integral in rectangular coordinates representing the volume of E. Do not evaluate the integral. (2)

(c) Again using symmetry, **set up** a triple integral in cylindrical coordinates representing the volume of E. Do not evaluate the integral. (2)

(d) **Set up** a triple integral in spherical coordinates representing the volume of E. Do not evaluate the integral. (4)

Question 6 [6]

Use a suitable change of variables to evaluate the integral

$$\iint_R \cos(x-y)\sin(2x+y)\ dx\,dy$$

where R is the parallelogram bounded by

$$x-y=0 \qquad x-y=\frac{\pi}{2} \qquad 2x+y=0 \qquad 2x+y=\pi.$$

Question 7 [5]

If C is the line segment connecting the point (a_1, b_1) to the point (a_2, b_2), show that

$$\int_C 2x\,dy - 2y\,dx = 2a_1b_2 - 2a_2b_1.$$

Question 8 [7]

Let $\mathbf{F}(x,y) = \left(3x^2 + y + \dfrac{x^2}{y^2}\right)\mathbf{i} + \left(x - \dfrac{2x^3}{3y^3} + 2\right)\mathbf{j}.$

(a) Using a systematic method, find a function f such that $\nabla f = \mathbf{F}$. Show **all** your work. (5)

(b) Use part (a) to evaluate $\displaystyle\int_C \left(3x^2 + y + \frac{x^2}{y^2}\right) dx + \left(x - \frac{2x^3}{3y^3} + 2\right) dy$, where C is the straight line $(1,1)$ to $(4,6)$. (2)

Question 9 [4]

Use Green's Theorem to evaluate $\displaystyle\oint_C \left(\cos x + y^2\right) dx + (\sin y + x)\ dy$, where C is the square consisting of the line segments from $(0,0)$ to $(2,0)$, from $(2,0)$ to $(2,2)$, from $(2,2)$ to $(2,0)$, and from $(2,0)$ to $(0,0)$.

Question 10 [3]

Show that there is no vector field $\mathbf{G} = \langle P, Q, R\rangle$ where P, Q and R have continuous second order partial derivatives such that

$$\text{curl }\mathbf{G} = \left\langle x, y^5, 4z^3\right\rangle.$$