

Lie Groups and Lie Algebras

Final Examination

APM8X07

Total 85 Marks

Question 1

Why is non-dimensionalisation important when solving ordinary or partial differential equations?

[10]

Question 2

Use non-dimensionalisation to simplify:

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = F \cos(\omega t).$$

Provide a physical interpretation to the parameters you obtain.

[10]

Question 3

Define a Hermitian matrix. Why are Hermitian matrices important in the study of Lie Groups and Lie Algebras?

[10]

Question 4

Use equilateral triangles to form a group of rotations. Give the algebra formed by the group. What is the name of the group formed by the rotations of an equilateral triangle?

[15]

Question 5

Consider the first-order system of equations $Y' = AY + B$ where Y is $n \times 1$, A is $n \times n$ and B is $n \times 1$. Use matrix exponentiation to show how you could solve the resulting system. **Do not solve the system.**

[10]

Question 6

Given that $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, determine $\exp(At)$.

[10]

Question 7

Discuss one real world application of Lie Groups and Lie Algebras.

[20]