

## Lie Groups and Lie Algebras

### Final Examination

APM8X07

Total Marks: 85

- All questions must be answered.
- The examination is open book.
- Timed Examination: This examination has a time limit of 2 hours.
- Timer Setting: The examination will be saved and submitted automatically when the time expires.
- Force Completion: Once started, this examination must be completed in one sitting. Do not leave the test before clicking Save and Submit.
- Multiple Attempts: This examination allows multiple attempts.
- Due Date: The examination starts at 8:30 and ends at 10:30.

#### Question 1

Why is non-dimensionalisation important when solving ordinary or partial differential equations?

[10]

#### Question 2

Use non-dimensionalisation to simplify:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F \cos(\omega t).$$

Provide a physical interpretation to the parameters you obtain.

[10]

### Question 3

Define a Unitary matrix. Why are Unitary matrices important in the study of Lie Groups and Lie Algebras?

[10]

### Question 4

Use squares to form a group of rotations.

Give the algebra formed by the group.

What is the name of the group formed by the rotations of a square, i.e. is the group orthogonal?

**You do not need to draw the squares, use the vertex labels (1 2 3 4) where the identity transformation can be written as (1 2 3 4, 1 2 3 4).**

[20]

### Question 5

Consider the first-order system of equations  $Y' = A Y$  where  $Y$  is  $n \times 1$  and  $A$  is  $n \times n$ , and  $Y(0) = Y_0$ .

Use matrix exponentiation to show how you could solve the differential equation.

**Do not solve the system.**

[10]

### Question 6

Given that  $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ , determine  $\exp(At)$ .

[10]

### Question 7

Discuss one real world application of Lie Groups and Lie Algebras.

[20]