University of Johannesburg
Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **ASME2A2**
LINEAR ALGEBRA FOR ENGINEERS

CAMPUS **APK**

EXAM **NOVEMBER 2021**

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Internal Examiner — Mr M Askes

30 Marks

Number of Pages: 2+2 Pages

Instructions:

1. Read all the questions carefully (from start to finish) before attempting to solve them.
2. Answer all questions.
3. Show all calculation and motivate all answers.
4. You must download the questions yourself and are not allowed to obtain them by any other means than via Blackboard. You are not allowed to share your downloaded copy of the questions with any other student. You MUST complete and submit the version allocated to you.
5. You will be allowed 3 opportunities to upload a file. Only the last file uploaded will be marked. Please make sure that you select the correct file. Please submit a single pdf file in which all pages have the same orientation.
6. Absolutely no teamwork or external assistance is allowed – if there is any indication that your work resembles another students work, or is not your own work, the case will be sent for analysis and a disciplinary hearing and possible expulsion may result.
7. The exam is open-book, so you may consult your textbook, slides, videos etc.
8. Please allow some time for uploading and submitting your solutions. That is, please do not leave it to the last minute to submit in case you find that you have connectivity issues.
9. Good luck!

Question 1 [5]

Given the sytsem of equations:

$$x + ky = 1$$
$$kx + y = 1$$

a) Find the values of k for which the system has

(i) no solution. (1)

(ii) exactly one solution. (1)

(iii) infinitely many solutions. (1)

b) Solve the system when it has exactly one solution. (2)

Question 2 [3]

Let $A = [a_{jk}]$ be the 3×3 matrix defined by

$$a_{jk} = \begin{cases} 0, & if \quad |j-k| > 1 \\ 1, & if \quad |j-k| = 1 \\ 2\cos x, & if \quad j = k \end{cases}$$

Find the acute angle x if $det(A) = 0$.

Question 3 [5]

Let $A = \begin{bmatrix} 0 & 0 \\ 2 & 6 \\ 1 & 2 \end{bmatrix}$.

Find matrices E,F,G and R such that E,F and G are elementary matrices, R is a matrix in row echelon form, and A = EFGR.

Question 4 [2]

Find the value of a if the angle between the vectors $(1, 0, -1, 3)$ and $(1, \sqrt{3}, 3, -3)$ in $\mathbb{R}^4$ is $a\pi$.

Question 5 [2]

Show that the following are not vector spaces:

a) Let V be the set of all real numbers. Define vector addition and scalar multiplication as follows:

$$\bar{x} + \bar{y} = the\ maximum\ of\ x\ and\ y$$
$$k\bar{x} = kx$$

(1)

b) Let V be the set of sequences, with usual addition but scalar multiplication defined as follows: For $\bar{u} = (u_1, u_2, \ldots u_k, \ldots)$ and c a scalar:

$$c\bar{u} = (cu_1 + c, cu_2 + c, \ldots cu_k + c, \ldots)$$

(1)

Question 6 [7]

(a) Let $P(2, 0, 3)$ be a point in $\mathbb{R}^3$ and let $\overline{v}_1 = (0, 0, -2)$ and $\overline{v}_2 = (-2, 4, 1)$ be two parallel noncolinear vectors in $\mathbb{R}^3$.

(i) Find the vector equation of the plane containing P, $\overline{v}_1$ and $\overline{v}_2$. (1)

(ii) Find parametric equations of the plane containing P, $\overline{v}_1$ and $\overline{v}_2$. (1)

(b) Let $\overline{u} = (0, 2, -2)$ and $\overline{v} = (1, 3, 0)$. Find a nonzero vector orthogonal to both $\overline{u}$ and $\overline{v}$. (2)

(c) If $\overline{u}$, $\overline{v}$ and $\overline{w}$ are vectors in $\mathbb{R}^n$, does $\overline{u} \bullet (\overline{v} \bullet \overline{w})$ exist? Explain. (1)

(d) Given that $\overline{u} \bullet (\overline{v} \times \overline{w}) = 3$, find $\overline{w} \bullet (\overline{v} \times \overline{u})$ (2)

Question 7 [4]

Let $x + 2z = 0$ be a plane in $\mathbb{R}^3$.

a) Find a basis for the plane. (3)

b) What is the dimesion of the plane? (1)

Question 8 [2]

M_{22} is the vector space of all 2×2 matrices, with the usual definitions of matrix addition and scalar multiplication.

Let $\overline{w}_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $\overline{w}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\overline{w}_3 = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$.
Is $\text{span}(\overline{w}_1, \overline{w}_2, \overline{w}_3) = M_{22}$? Motivate your answer.