



DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **ASMAV1A**
ENGINEERING MATHEMATICS V1A

CAMPUS **DFC**
ASSESSMENT **NOVEMBER EXAM**

DATE 08/11/2021

TIME 16:30

ASSESSOR

MR M SAGMING

INTERNAL MODERATOR

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DURATION 3 HOURS

MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 5 PAGES

INSTRUCTIONS:

- 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.**
- 2. CALCULATORS AND THREE DECIMAL PLACES ALLOWED.**
- 3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.**
- 4. DOWNLOAD AND COMPLETE AT HOME, SCAN AND UPLOAD A SINGLE PDF OF YOUR SOLUTION ON BLACKBOARD.**
- 5. ADHERE TO THE TERMS OF THE HONESTY DECLARATION.**

Question 1 [20 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The exact value of $\arctan(-1)$ is:

[2]

(a) $\frac{\pi}{4}$

(b) -1

(c) $3\frac{\pi}{4}$

(d) $-\frac{\pi}{4}$

(e) None of the above.

1.2 Given $f'(14) = 15$, $f'(2) = 11$, $g(14) = 2$, $g'(14) = 5$ and $F(x) = f(g(x))$. What is the value of $F'(14)$? [2]

(a) 24

(b) 17

(c) 55

(d) 20

(e) None of the above.

1.3 Suppose $H = f \circ g \circ h$ and $H(x) = \sqrt[10]{\sqrt{x} - 3}$. Select the correct option below:

[2]

(a) $g(x) = \sqrt[10]{x}$

(b) $h(x) = \sqrt[10]{x}$

(c) $f(x) = \sqrt[10]{x}$

(d) All of the above.

(e) None of the above.

1.4 Find the vertex point (h, k) of the quadratic function $f(x) = 2ax^2 + ax - a$. [2]

(a) $\left(-\frac{1}{4}, -\frac{9}{8}a\right)$

(b) $\left(-\frac{1}{4}, -\frac{9}{16}\right)$

(c) $\left(\frac{1}{4}, -\frac{9}{8}a\right)$

(d) $\left(\frac{1}{4}, -\frac{9}{16}a\right)$

(e) None of the above

1.5 $\cosh(\ln(x)) =$ [2]

(a) $\frac{1-x^2}{2x}$

(b) $\frac{2x}{x^2-1}$

(c) $\frac{x^2+1}{2x}$

(d) $\frac{x^2-1}{2x}$

(e) None of the above.

1.6 Which of the following functions can be differentiated **without** using the Chain Rule? [2]

(i) $y = \tan(7x^2 + 2)$ (ii) $y = \frac{x}{x+1}$ (iii) $y = \sqrt{x} \sec x$ (iv) $y = \sqrt{x \cos x}$

(a) (i) and (ii) only

(b) (ii) and (iii) only

(c) (i) and (iv) only

(d) (i), (ii) and (iv) only

(e) None of the above.

1.7 The point $P(-3, -8)$ is on the graph of $y = f(x)$. Which point will be on the graph of $y = -f(x-5)$? [2]

(a) $(2, 8)$

(b) $(-8, 8)$

(c) $(-8, -8)$

(d) $(8, -8)$

(e) None of the above.

1.8 The value of K so that the function f is continuous at the number $a = 1$ is: [2]

$$f(x) = \begin{cases} K - x^2 & \text{if } x \leq 1 \\ \ln\left(\frac{e^2}{x}\right) & \text{if } x > 1 \end{cases}$$

- (a) 1
- (b) 3
- (c) 2
- (d) $\ln 2$
- (e) None of the above.

1.9 The value of K such that $\int_1^K \frac{x + x^3}{x^2} dx = \frac{3 + 2 \ln 2}{2}$ is: [2]

- (a) 4
- (b) 2
- (c) $\frac{3}{2}$
- (d) $2 \ln 2$
- (e) None of the above.

1.10 The solution to $\lim_{x \rightarrow -\infty} (2 \tan^{-1}(cx))$, for $c < 0$ is: [2]

- (a) π
- (b) ∞
- (c) $-\frac{\pi}{2}$
- (d) $-\infty$
- (e) None of the above.

Question 2 [3 marks]

Prove the identity: $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$.

Question 3 [5 marks]

Use **Cramer's Rule** to solve the following system of linear equations:

$$x + y + z = 6$$

$$2y + 5z = -4$$

$$2x + 5y - z = 27$$

Question 4 [6 marks]

(a) Determine whether $f(x) = \frac{1}{\tan x}$ is even, odd or neither. [2]

(b) State the domain and the range of the function shown below using interval notation: [4]

$$y = \frac{\pi}{2} + \arctan \theta.$$

Question 5 [3 marks]

Solve for x if $2^{\log(x+1)} = \sin \frac{\pi}{2}$.

Question 6 [6 marks]

Consider the function: $f(x) = \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

(a) Find the vertical asymptote(s). [2]

(b) Find the horizontal asymptote(s). [4]

Question 7 [5 marks]

Consider the function below:

$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ 4 - x & \text{if } 1 < x \leq 4 \\ -1 & \text{if } x > 4 \end{cases}$$

Use limits to show that f is continuous at $x = 1$.

Question 8 [3 marks]

Compute $\lim_{x \rightarrow \infty} \frac{2 - \cos x}{x + 3}$ using the **Squeeze Theorem**.

Question 9 [9 marks]

(a) If $y = (\sqrt{x})^{\sqrt{x}}$, find $\frac{dy}{dx}$ using **Logarithmic Differentiation**. [4]

(b) Find the equation of the tangent line to the curve at the given point: [5]

$$x^4 + 4xy + y^4 = 6, (1, 1).$$

Question 10 [5 marks]

Use **Binomial Theorem** to expand: $\left(x^2 - \frac{3}{y}\right)^5$

Question 11 [5 marks]

Evaluate the indefinite integral: $\int x^3 \sqrt{x^2 + K} \, dx, \quad K \geq 0.$