1 Exam [38]

Instructions:
Please answer the questions carefully, and clearly write your student number and name.
When uploading your exam, upload the document as a pdf.
When saving your document, please save the file with your student number then name (such as "201912345-JohnPaul.pdf").
Note that the deadline to submit the exam is 11:30 on Wednesday 23 June 2021.
Additionally be aware that there are 11 questions and 3 pages of this exam.

Question 1 [3]

Find the domain and range of the following functions:

(a) $f(x, y) = \sqrt{2x^2 + y}$.

(b) $f(x, y) = e^{\ln(x+y^2)}$.

(c) $f(x, y, z) = \frac{x}{z}$.

Question 2 [4]

Find the following limits if they exist, otherwise show that the limit does not exist.

(a)

$$\lim_{(x,y)\to(1,2)} \frac{(x-1)(y-2)}{\sqrt{(x-1)^2 + (y-2)^2}}$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x - 4y}{6y + 7x}$$

Question 3 [3]

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the following equation:

$$y^3x^2 + 2\ln(xy) - \cos^2(x) = x\sin(y) + 2.$$

Question 4 [4]

Determine f_{uu} if

$$f = f(x, y) \qquad x = u^2 + 3v \qquad y = uv.$$

Assume that all second order derivatives are continuous.

Question 5 [3]

Evaluate the integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, \mathrm{d}x\, \mathrm{d}y.$$

Question 6 [4]

Let E be the region between the two planes $x + y + z = 2$ and $z = 0$ and inside the cylinder $y^2 + x^2 = 1$. By making use of cylindrical coordinates, setup (but do not evaluate) the triple integral

$$\iiint_E x\, \mathrm{d}V.$$

Question 7 [4]

Use spherical coordinates to setup (but not evaluate) the integral to calculate the volume of the region outside the sphere $x^2 + y^2 + (z-1)^2 = 1$ and inside the hemisphere $x^2 + y^2 + z^2 = 4$ with $z \geq 0$.

Question 8 [4]

Let R be the region bounded by $xy = 1$, $xy = 3$, $y = 2$ and $y = 6$. Evaluate

$$\iint_R xy^3\, \mathrm{d}A$$

by making use of the transformation

$$x = \frac{v}{6u} \quad \text{and} \quad y = 2u.$$

Question 9 [2]

Let $\mathbf{F}$ be a conservative vector field. Compute

$$\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

where C is the ellipse given by

$$\frac{(x-5)^2}{4} + \frac{y^2}{9} = 1,$$

with the counter clockwise rotation.

Question 10 [4]

Use Green's Theorem to evaluate

$$\int_C yx^2 \, \mathrm{d}x \; - x^2 \, \mathrm{d}y$$

where C is the path starting at $(0, -5)$ going along the line $y = 0$ to the point $(0, 5)$ then traveling along the circle $x^2 + y^2 = 25$ where $x \leq 0$ back to the point $(0, -5)$.

Question 11 [3]

Determine if the following vector field is conservative:

$$\mathbf{F} = \left\langle 4y^2 + \frac{3x^2y}{x^2} \, , \; 8xy + \frac{x^3}{z^2} \, , \; 11 - \frac{2x^3y}{z^3} \right\rangle .$$