UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE   MAT01A2/MAT2A10
                SEQUENCES, SERIES AND VECTOR CALCULUS

CAMPUS   APK
ASSESSMENT   EXAMINATION WRITTEN

DATE 07/06/2021   TIME 12:30

ASSESSOR(S)   DR A SWARTZ

INTERNAL MODERATOR   DR A GOSWAMI

DURATION 100 MINUTES   MARKS 25

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 1 PAGES

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
              2. NO CALCULATORS ARE ALLOWED.
              3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.

Question 1 [2 marks]

Prove or disprove the following statement:

If $\sum a_n$ and $\sum b_n$ are both divergent then $\sum (a_n + b_n)$ is divergent. Justify your reasoning completely.

Question 2

Test the following series for convergence or divergence. Justify your reasoning.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{n^2}$ (2)

(b) $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{3} - 1)^n$ (3)

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 2n}{1 + n^2}$ (3)

Question 3 [4 marks]

Find the Maclaurin series for f and its radius of convergence:

$$f(x) = (1 - 3x)^{-6}.$$

Question 4 [2 marks]

Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t^9\mathbf{i} + 4t^3\mathbf{j} - t^6\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j}$.

Question 5 [3 marks]

A particle moves with position function $\mathbf{r}(t) = 4\sqrt{2}t\mathbf{i} - e^{4t}\mathbf{j} + e^{4t}\mathbf{k}$. Find the acceleration of the particle.

Question 6 [2 marks]

Find the integral $\displaystyle\int (\cos 7t\mathbf{i} + \sin 7t\mathbf{j} + e^{-t/5}\mathbf{k})\, dt$.

Question 7 [4 marks]

Find the vectors $\mathbf{T}$ and $\mathbf{N}$ at the given point:

$$\mathbf{r}(t) = \langle \cos t, \sin t, \ln \sin t \rangle; \qquad \langle 0, 1, 0 \rangle.$$