



FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **MAT01A2/MAT2A10**
SEQUENCES, SERIES AND VECTOR CALCULUS

CAMPUS **APK**
ASSESSMENT **EXAMINATION**

DATE 07/06/2021

TIME 08:30

ASSESSOR(S)

DR A SWARTZ

INTERNAL MODERATOR

DR A GOSWAMI

DURATION 80 MINUTES

MARKS 15

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 13 PAGES

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE
ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [15 marks]**Question 1.1**

(1)

QUESTION1.1.1 A sequence $\{a_n\}$ is defined recursively as $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$, and $a_1 = a_2 = 14$. Find the limit of the sequence. (1)

- a) 14
- b) 6
- c) 26
- d) 1
- e) None of the these is correct

QUESTION1.1.2 A sequence $\{a_n\}$ is defined recursively as $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$, and $a_1 = a_2 = 16$. Find the limit of the sequence. (1)

- a) 16
- b) 6
- c) 26
- d) 1
- e) None of the these is correct

QUESTION1.1.3 A sequence $\{a_n\}$ is defined recursively as $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$, and $a_1 = a_2 = 18$. Find the limit of the sequence. (1)

- a) 18
- b) 6
- c) 26
- d) 1
- e) None of the these is correct

Question 1.2

(1)

QUESTION1.2.1 Determine whether the sequence defined by $a_n = \frac{n^2 - 5}{6n^2 + 1}$ converges or diverges. If it converges, find its limit. (1)

- a) $\frac{1}{6}$
- b) -5
- c) $\frac{-5}{6}$
- d) diverges.
- e) None of the these is correct

QUESTION1.2.2 Determine whether the sequence defined by $a_n = \frac{n^2 - 5}{7n^2 + 1}$ converges or diverges. If it converges, find its limit. (1)

- a) $\frac{1}{7}$
- b) -5
- c) $\frac{-5}{7}$
- d) diverges.
- e) None of the these is correct

QUESTION1.2.3 Determine whether the sequence defined by $a_n = \frac{n^2 - 5}{8n^2 + 1}$ converges or diverges. If it converges, find its limit. (1)

- a) $\frac{1}{8}$
- b) -5
- c) $\frac{-5}{8}$
- d) diverges.
- e) None of the these is correct

Question 1.3

(2)

QUESTION1.3.1 Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-8)^n}{\sqrt{n}}$$

- a) $R = 1, I = (7, 9]$
- b) $R = 1, I = [7, 9)$
- c) $R = 8, I = [-8, 8)$
- d) $R = 8, I = (-8, 8)$
- e) None of the these is correct

QUESTION1.3.2 Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-6)^n}{\sqrt{n}}$$

- a) $R = 1, I = (5, 7]$
- b) $R = 1, I = [5, 7)$
- c) $R = 6, I = (5, 7)$
- d) $R = 6, I = [5, 7]$
- e) None of the these is correct

QUESTION1.3.3 Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-4)^n}{\sqrt{n}}$$

- a) $R = 1, I = (3, 5]$
- b) $R = 1, I = [3, 5)$
- c) $R = 4, I = (3, 5)$
- d) $R = 4, I = [3, 5]$
- e) None of the these is correct

Question 1.4

(1)

QUESTION1.4.1 Determine whether the series converges or diverges. If it converges, determine its sum.

$$\sum_{n=0}^{\infty} 3^n 4^{-n+1}$$

- a) 16
- b) diverges
- c) 3
- d) 12
- e) No answer here is correct

QUESTION1.4.2 Determine whether the series converges or diverges. If it converges, determine its sum.

$$\sum_{n=0}^{\infty} 5^n 6^{-n+1}$$

- a) 36
- b) 30
- c) 3
- d) 5
- e) diverges
- f) No answer here is correct

QUESTION1.4.3 Determine whether the series converges or diverges. If it converges, determine its sum.

$$\sum_{n=0}^{\infty} 6^n 7^{-n+1}$$

- a) 49
- b) 36
- c) 3
- d) 5
- e) diverges
- f) No answer here is correct

Question 1.5

(2)

QUESTION1.5.1 Find the Maclaurin series for $f(x)$ using the definition of the Maclaurin series.

$$f(x) = x \cos(4x)$$

- a) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$
- b) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+1}}{(2n)!}$
- c) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$
- d) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(n)!}$
- e) None of these is correct

QUESTION1.5.2 Find the Maclaurin series for $f(x)$ using the definition of the Maclaurin series.

$$f(x) = x \cos(3x)$$

- a) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+1}}{(2n)!}$
- b) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{(2n)!}$
- c) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n}}{(2n)!}$
- d) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+1}}{(n)!}$
- e) None of these is correct

QUESTION1.5.3 Find the Maclaurin series for $f(x)$ using the definition of the Maclaurin series.

$$f(x) = x \cos(5x)$$

- a) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n} x^{2n+1}}{(2n)!}$
- b) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^n x^{2n+1}}{(2n)!}$
- c) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n} x^{2n}}{(2n)!}$
- d) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n} x^{2n+1}}{(n)!}$
- e) None of these is correct

Question 1.6

(2)

QUESTION1.6.1 A rubber ball is dropped from a height of 8 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels.

- a) 24
- b) 16
- c) 36
- d) 32
- e) none of these answers is correct

QUESTION1.6.2 A rubber ball is dropped from a height of 16 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels.

- a) 48
- b) 24
- c) 36
- d) 96
- e) none of these answers is correct

QUESTION1.6.3 A rubber ball is dropped from a height of 32 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels.

- a) 96
- b) 48
- c) 64
- d) 132
- e) none of these answers is correct

Question 1.7

(1)

QUESTION1.7.1 Find the limit

$$\lim_{t \rightarrow 0^+} \langle 8 \cos t, 24 \sin t, 5t \ln t \rangle$$

- a) $\mathbf{r}(t) = 8\mathbf{i}$
- b) $\mathbf{r}(t) = 8\mathbf{i} + 24\mathbf{j} + 5\mathbf{k}$
- c) $\mathbf{r}(t) = 8\mathbf{k}$
- d) $\mathbf{r}(t) = 8\mathbf{i} - 5\mathbf{k}$
- e) none of these answers is correct

QUESTION1.7.2 Find the limit

$$\lim_{t \rightarrow 0^+} \langle 8 \cos t, 5t \ln t, 24 \sin t \rangle$$

- a) $\mathbf{r}(t) = 8\mathbf{i}$
- b) $\mathbf{r}(t) = 8\mathbf{i} + 24\mathbf{j} + 5\mathbf{k}$
- c) $\mathbf{r}(t) = 8\mathbf{k}$
- d) $\mathbf{r}(t) = 8\mathbf{i} - 5\mathbf{k}$
- e) none of these answers is correct

QUESTION1.7.3 Find the limit

$$\lim_{t \rightarrow 0^+} \langle 5t \ln t, 8 \cos t, 24 \sin t \rangle$$

- a) $\mathbf{r}(t) = 8\mathbf{j}$
- b) $\mathbf{r}(t) = 8\mathbf{i} + 24\mathbf{j} + 5\mathbf{k}$
- c) $\mathbf{r}(t) = 8\mathbf{k}$
- d) $\mathbf{r}(t) = 8\mathbf{i} - 5\mathbf{k}$
- e) none of these answers is correct

Question 1.8

(1)

QUESTION1.8.1 Find the unit tangent vector for the curve given by

$$\mathbf{r}(t) = \left\langle \frac{1}{5}t^5, \frac{1}{3}t^3, t \right\rangle.$$

- a) $\frac{\langle t^4, t^2, 1 \rangle}{\sqrt{t^8 + t^4 + 1}}$
- b) $\frac{\langle t^4, t^2, 1 \rangle}{\sqrt{t^{10} + t^4}}$
- c) $\frac{\langle t^4, t^2, 1 \rangle}{\sqrt{4t^8 + 4t^4 + 1}}$
- d) $\frac{\langle t^4, t^2, 1 \rangle}{\sqrt{t^8 + t^4}}$
- e) none of these answers is correct

QUESTION1.8.2 Find the unit tangent vector for the curve given by

$$\mathbf{r}(t) = \left\langle \frac{1}{6}t^6, \frac{1}{4}t^4, t \right\rangle.$$

- a) $\frac{\langle t^5, t^3, 1 \rangle}{\sqrt{t^{10} + t^6 + 1}}$
- b) $\frac{\langle t^5, t^3, 1 \rangle}{\sqrt{t^{10} + t^6}}$
- c) $\frac{\langle t^5, t^3, 1 \rangle}{\sqrt{4t^{10} + 4t^9 + 1}}$
- d) $\frac{\langle t^5, t^3, 1 \rangle}{\sqrt{t^{10} + t^4}}$
- e) none of these answers is correct

QUESTION1.8.3 Find the unit tangent vector for the curve given by

$$\mathbf{r}(t) = \left\langle \frac{1}{7}t^7, \frac{1}{5}t^5, t \right\rangle.$$

- a) $\frac{\langle t^6, t^4, 1 \rangle}{\sqrt{t^{12} + t^8 + 1}}$
- b) $\frac{\langle t^6, t^4, 1 \rangle}{\sqrt{t^{12} + t^8}}$
- c) $\frac{\langle t^6, t^4, 1 \rangle}{\sqrt{4t^{12} + 4t^8 + 1}}$
- d) $\frac{\langle t^6, t^4, 1 \rangle}{\sqrt{t^{12} + t^{10}}}$
- e) none of these answers is correct

Question 1.9

(2)

QUESTION1.9.1 Find the length of the curve $\mathbf{r}(t) = -3t \mathbf{i} + 2t \mathbf{j} - t \mathbf{k}$, $-2 \leq t \leq 1$.

- a) $3\sqrt{14}$
- b) $2\sqrt{14}$
- c) $6\sqrt{14}$
- d) $\sqrt{14}$
- e) none of these answers is correct

QUESTION1.9.2 Find the length of the curve $\mathbf{r}(t) = 3t \mathbf{i} - 2t \mathbf{j} + t \mathbf{k}$, $-2 \leq t \leq 1$.

- a) $3\sqrt{14}$
- b) $2\sqrt{14}$
- c) $6\sqrt{14}$
- d) $\sqrt{14}$
- e) none of these answers is correct

QUESTION1.9.3 Find the length of the curve $\mathbf{r}(t) = t \mathbf{i} - 2t \mathbf{j} + 3t \mathbf{k}$, $-2 \leq t \leq 1$.

- a) $3\sqrt{14}$
- b) $2\sqrt{14}$
- c) $6\sqrt{14}$
- d) $\sqrt{14}$
- e) none of these answers is correct

Question 1.10

(2)

QUESTION1.10.1 What is the curvature of $\mathbf{r}(t) = \langle \sqrt{13}t, e^t, e^{-t} \rangle$ at the point $(0, 1, 1)$?

- a) $\frac{\sqrt{2}}{15}$
- b) $\sqrt{15}$
- c) $15\sqrt{2}$
- d) $\frac{15}{\sqrt{2}}$
- e) none of these answers is correct

QUESTION1.10.2 What is the curvature of $\mathbf{r}(t) = \langle e^t, \sqrt{13}t, e^{-t} \rangle$ at the point $(1, 0, 1)$?

- a) $\frac{\sqrt{2}}{15}$
- b) $\sqrt{15}$
- c) $15\sqrt{2}$
- d) $\frac{15}{\sqrt{2}}$
- e) none of these answers is correct

QUESTION1.10.3 What is the curvature of $\mathbf{r}(t) = \langle e^t, e^{-t}, \sqrt{13}t \rangle$ at the point $(1, 1, 0)$?

- a) $\frac{\sqrt{2}}{15}$
- b) $\sqrt{15}$
- c) $15\sqrt{2}$
- d) $\frac{15}{\sqrt{2}}$
- e) none of these answers is correct