UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACHELOR OF ENGINEERING *TECHNOLOGY: CIVIL*
SUBJECT	:	**STRUCTURAL ANALYSIS GP2 3B**
CODE	:	**STRCIB3**
DATE	:	FINAL EXAMINATION 14 NOVEMBER 2019
DURATION	:	(SESSION 1) 08:30 - 11:30
WEIGHT	:	40 : 60
TOTAL MARKS	:	100

ASSESSOR	:	Mr S Ngidi
MODERATOR	:	Mr SM Rakgate
NUMBER OF PAGES	:	3 PAGES

INSTRUCTIONS	:	ONLY ONE POCKET CALCULATOR PER CANDIDATE MAY BE USED.
REQUIREMENTS	:	2 ANSWER BOOKLETS

INSTRUCTIONS TO STUDENTS

PLEASE ANSWER ALL QUESTIONS.
SHOW ALL THE STEPS FOR CALCULATIONS CLEARLY.

QUESTION 1 [30]

Construct the influence lines for the reactions at C and D, shear at H and the moment at G of the beam shown in Figure Q1. Hence, calculate the positive maximum value of each case due to a uniform live load of 500kN/m and a single moving concentrated live load of 250kN. (30)

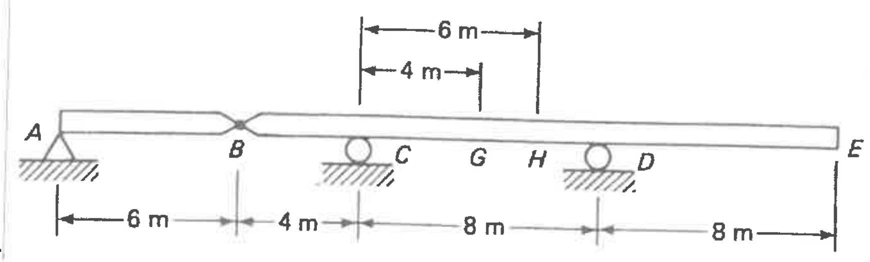


Figure Q1

QUESTION 2 [40]

a) The frame in Figure Q2a is pinned at A, fixed at D, roller supported at C and has a rigid joint B. Produce the full three part qualitative analysis solution for the frame due to the applied point load.

(10)

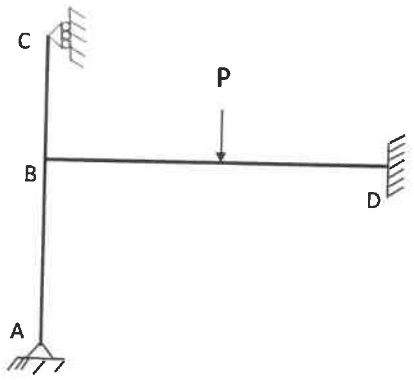


Figure Q2a

b) The beam shown in Figure Q2b is pinned at A, and has roller supports at B and C. A load of 80 kN is applied at D and at E. Determine the reactions, using flexibility method. Hence draw the shear and bending moment diagrams for the beam.

(30)

($E = 200x10^6$ kN/m² and $I = 160x10^{-6}$ m⁴)

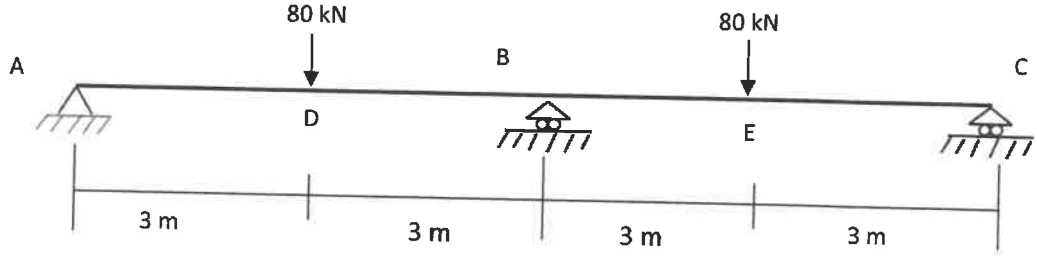


Figure Q2b

QUESTION 3 [30]

Determine the global stiffness matrix for the truss shown in Figure Q3, by **method of direct computation**.

(30)

(A = 4000 mm^2, E = 210 GPa)

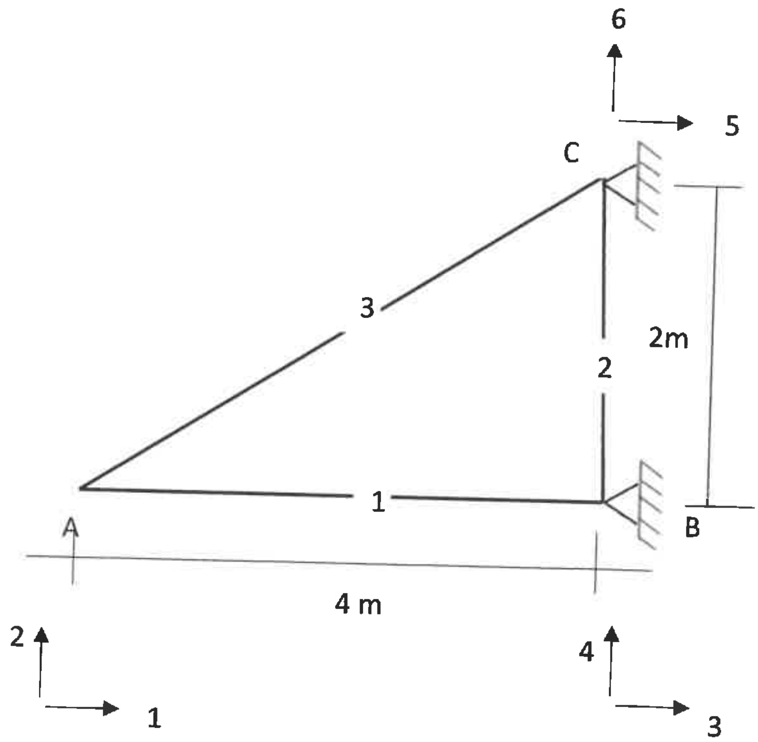


Figure Q3

STRUCTURAL ANALYSIS 3B

EXAM MEMO

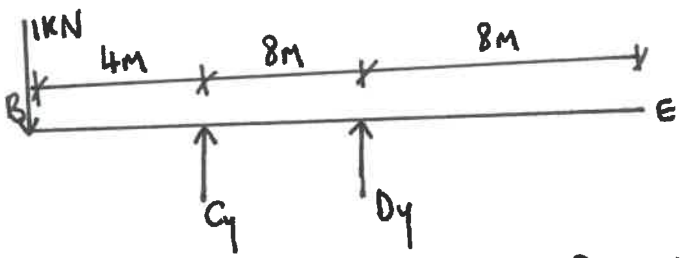
QUESTION 1

Influence lines for reaction at C and D

Unit load at A:

$C_y = 0$ & $D_y = 0$

Unit load at B:



$\circlearrowleft + \Sigma M_c = 0;$ $-1(4) - D_y(8) = 0;$ $D_y = -0.5\,kN$

$\circlearrowleft + \Sigma M_D = 0;$ $-1(12) + C_y(8) = 0;$ $C_y = 1.5\,kN$

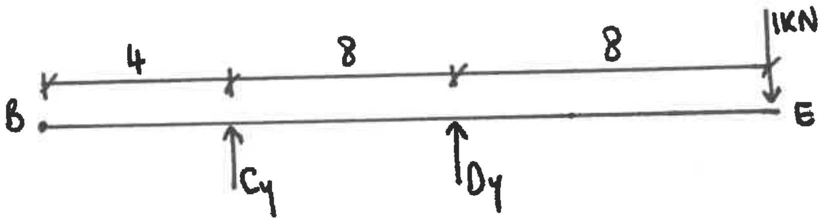
Unit load at C:-

$C_y = 1KN$, $D_y = 0kN$

Unit load at D:-

$D_y = 1KN$, $C_y = 0kN$

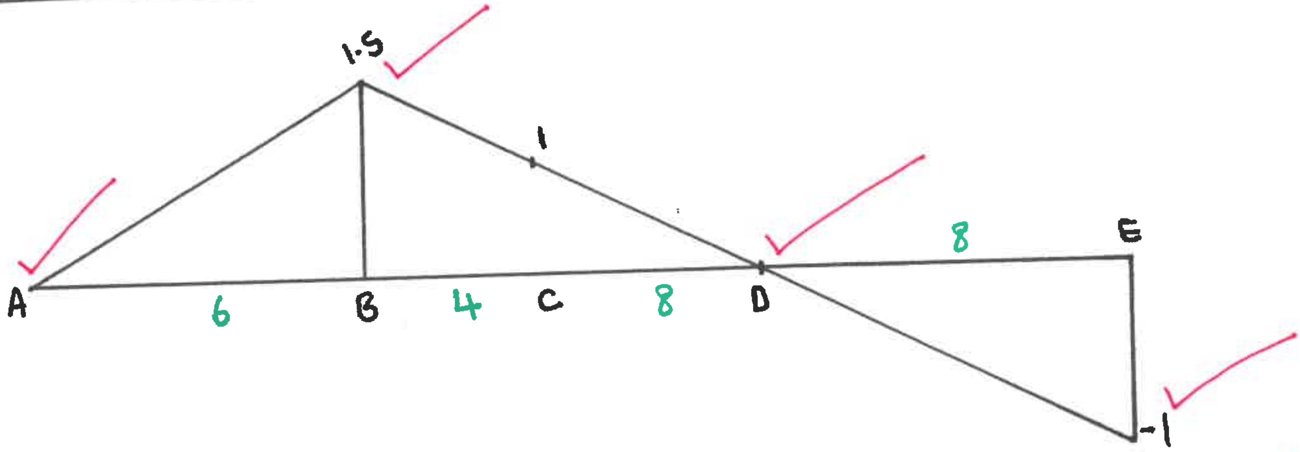
Unit load at E :-



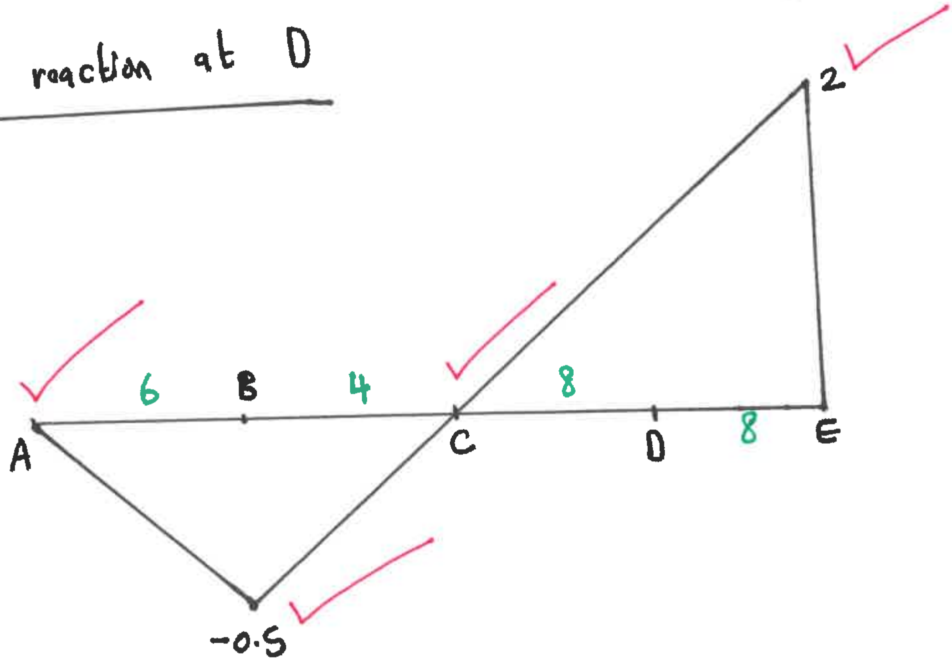
$\circlearrowleft + \Sigma M_C = 0;\quad -D_y(8) + 1(16) = 0 \quad ; \quad D_y = 2\,kN$

$\circlearrowleft + \Sigma M_D = 0;\quad C_y(8) + 1(8) = 0 \quad ; \quad C_y = -1\,kN$

Influence Line for the reaction at C



Influence Line for the reaction at D

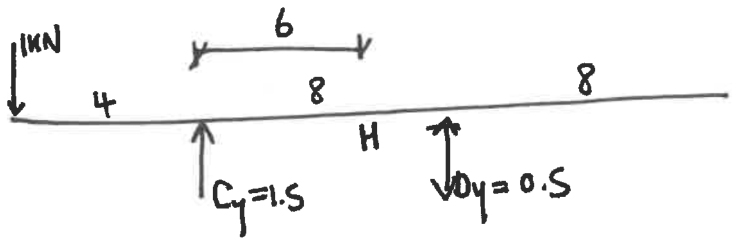


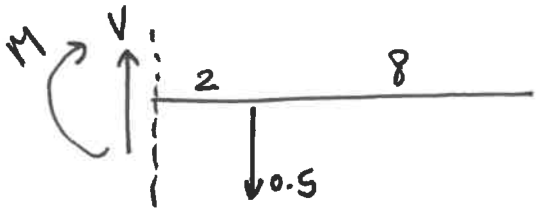
Shear at H

Unit load at A :-

$V_H = 0kN$

Unit load at B :-





$+\uparrow \Sigma F_y = 0;$ $V - 0.5 = 0$

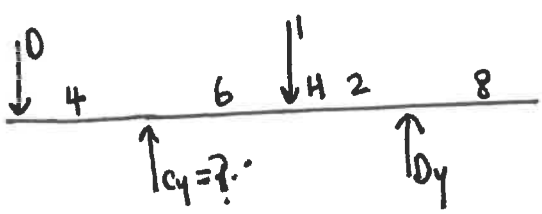
$\therefore V_H = 0.5kN$

Unit load at C :-

$C_y = 1$ and $D_y = 0kN$; $\therefore V_H = 0kN$

Unit load at H :-

Unit load at left of H



$\circlearrowright + \Sigma M_c = 0;$ $1(6) - D_y(8) = 0$

$D_y = 0.75kN$

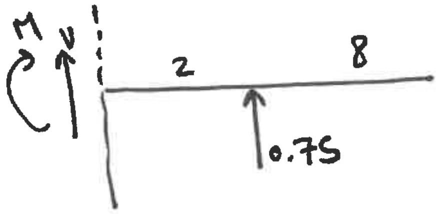
$\circlearrowright + \Sigma M_D = 0;$ $-1(2) + C_y(8) = 0$

$C_y = 0.25$

Unit load at left of H

$+\uparrow \Sigma F_y = 0; \quad V_H + 0.75 = 0$

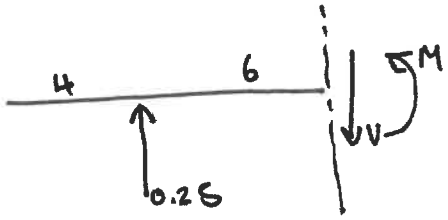
$V_H = -0.75$



Unit load at right of H

$+\uparrow \Sigma F_y = 0; \quad 0.25 - V = 0$

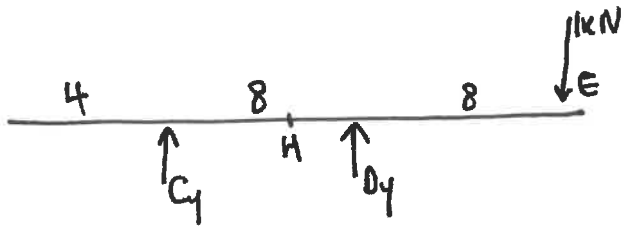
$V_H = 0.25$



Unit load at D:

$D_y = 1, \quad C_y = 0, \quad \therefore V_H = 0$

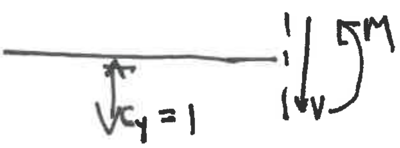
Unit load at E



$\circlearrowleft + \Sigma M_C = 0; \quad -D_y(8) + 1(16) = 0 \quad ; \quad D_y = 2\text{kN}$ ✓

$\circlearrowleft + \Sigma M_D = 0; \quad C_y(8) + 1(8) = 0 \quad ; \quad C_y = -1\text{kN}$ ✓

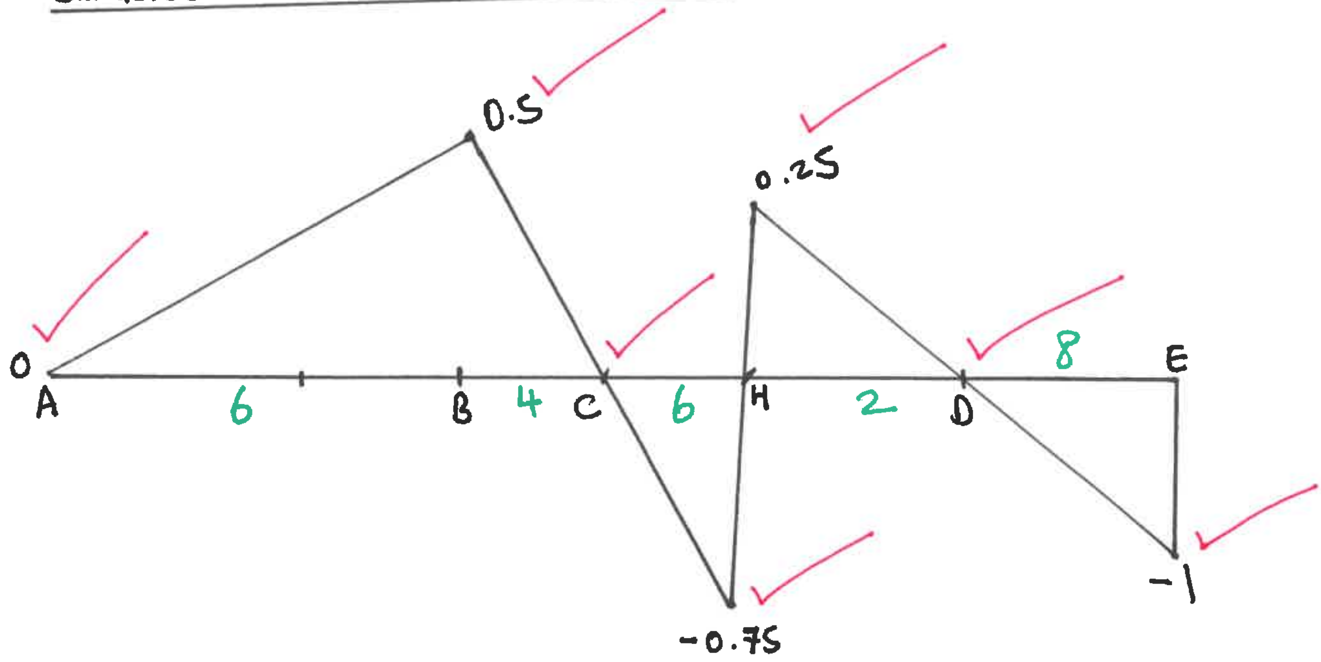
Now;



$+\uparrow \Sigma F_y = 0; \quad -1 - V = 0$

$V_H = -1\text{kN}$

Influence line for shear at H



Moment at G

Unit load at A:

$M_G = 0$

Unit load at B:

$\circlearrowright + \Sigma M = 0;$

$M + 0.5(4) = 0$

$M_G = -2 \, KN.M$

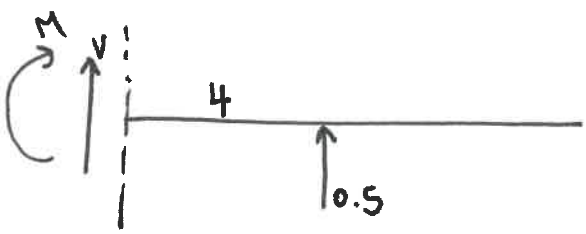
Unit load at C:

$C_y = 1KN$, $M_G = 0 \, KN.M$

Unit load at D :

$D_y = 1KN$, $M_G = 0 KN.M$

Unit load at G:



$\circlearrowright + \Sigma M = 0;$

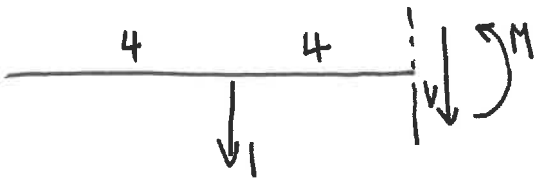
$M - 0.5(4) = 0$

$M = 2 \, KN.M$

Unit load at D :-

$D_y = 1KN$, $M_G = 0$

Unit load at E:

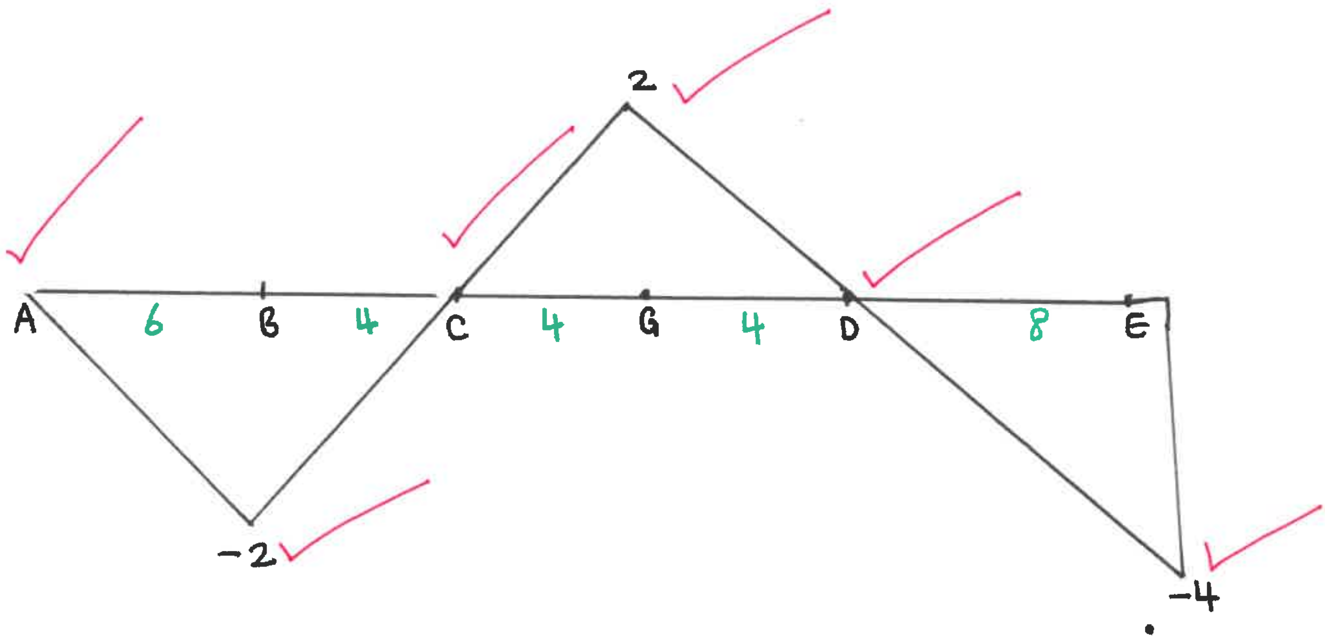


$\circlearrowleft + \Sigma M = 0;$

$-M - 1(4) = 0$

$M = -4 \text{ kN}\cdot\text{M}$

Influence Line for moment at G



Maximum reaction at C & D

$$(C_y)_{max} = 500\left(\tfrac{1}{2} \times 18 \times 1.5\right) + 250(1.5) = 7125 \text{ kN}$$

$$(D_y)_{max} = 500\left(\tfrac{1}{2} \times 16 \times 2\right) + 250(2) = 8500 \text{ kN}$$

Maximum shear at H

$$V_{max} = 500\left(\tfrac{1}{2} \times 10 \times 0.5\right) + 500\left(\tfrac{1}{2} \times 2 \times 0.25\right) + 250(0.5) = 1500 \text{ kN}$$

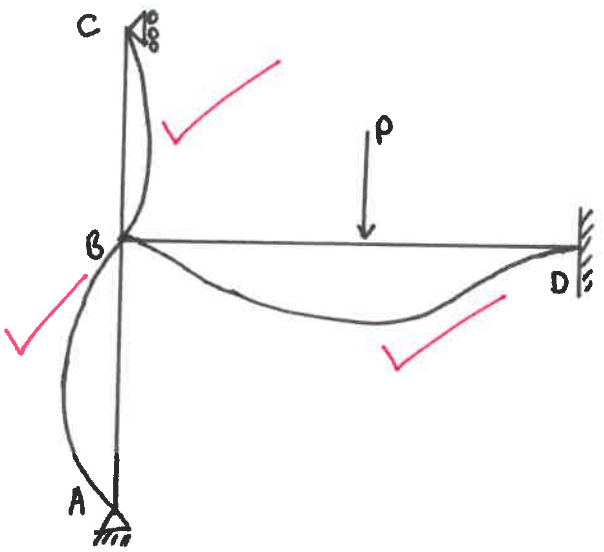
Maximum moment at G

$$M_{max} = 500\left(\tfrac{1}{2} \times 8 \times 2\right) + 250(2) = 4500 \text{ kN.m}$$

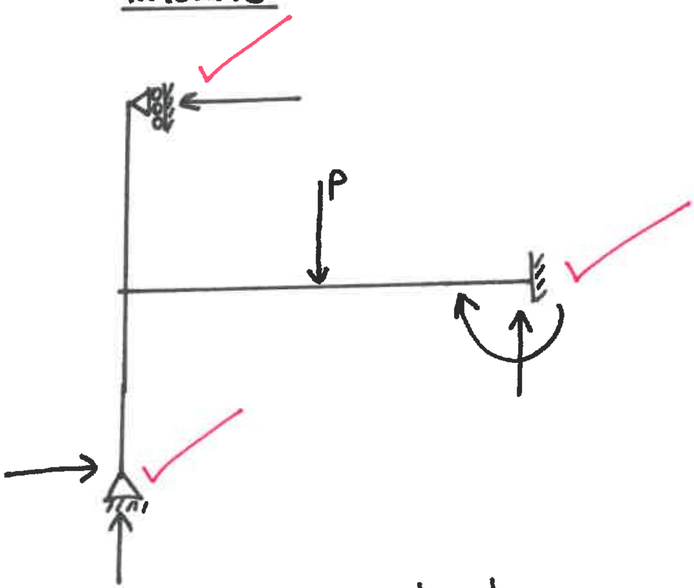
QUESTION 2

(a)

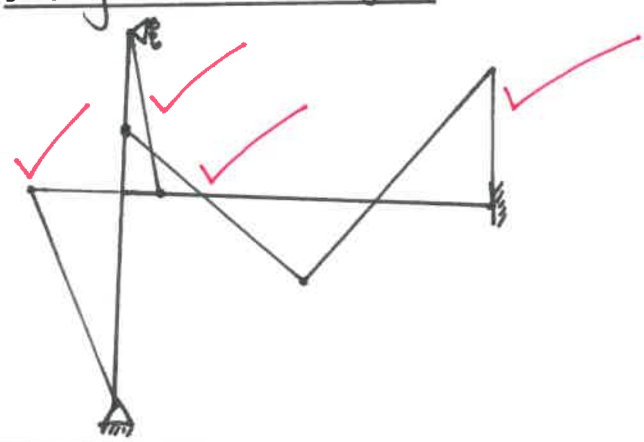
Deflected Shape



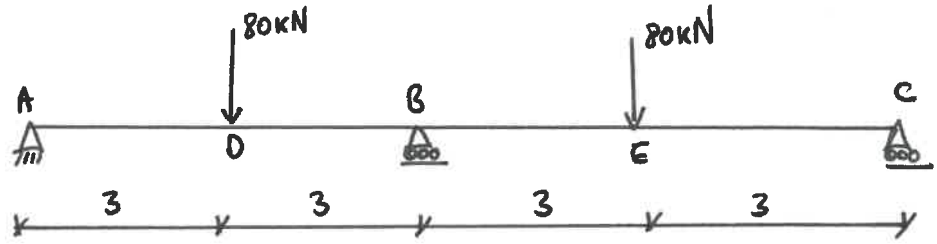
Reactions



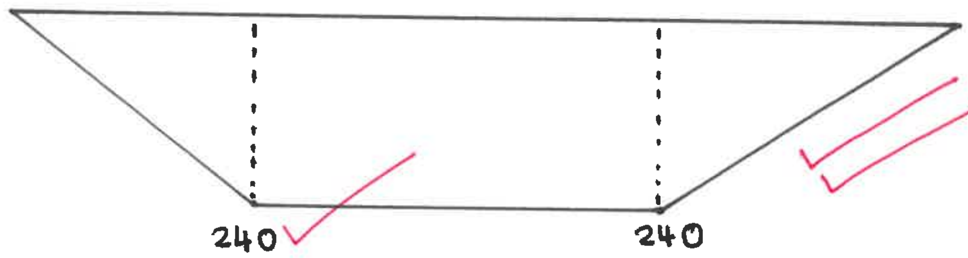
Bending Moment diagram



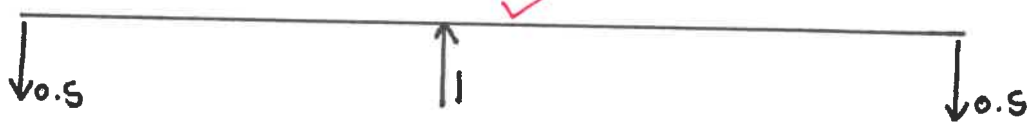
(b)

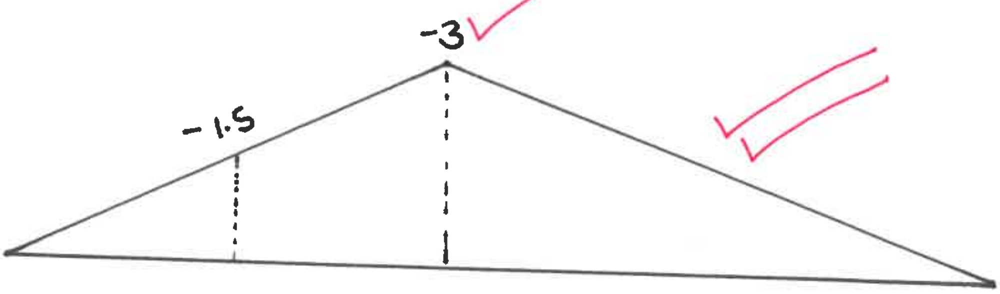


Remove V_B



Real Moment - M





Virtual moment - m

$$\Delta_1 = \int \frac{M m}{EI} dL$$

$$\Delta_1 = \left[\tfrac{1}{2} \times 3 \times 240 \times (-1)\right] \times 2 + \left[3 \times 240 \times (-2.25)\right] \times 2$$

$$\Delta_1 = \frac{-720}{EI} - \frac{3240}{EI} = \frac{-3960}{EI} = \frac{-3960}{200 \times 10^6 \times 160 \times 10^{-6}} = -\frac{99}{800}$$

$$\delta_{11} = \int \frac{M^2}{EI} dL = 2\left[\tfrac{1}{2} \times 6 \times (-3) \times (-2)\right] = \frac{36}{EI}$$

$$= \frac{36}{200 \times 10^6 \times 160 \times 10^{-6}} = \frac{9}{8000}$$

Compatibility Eq:

$$\delta_{11} X_1 + \Delta_1 = 0$$

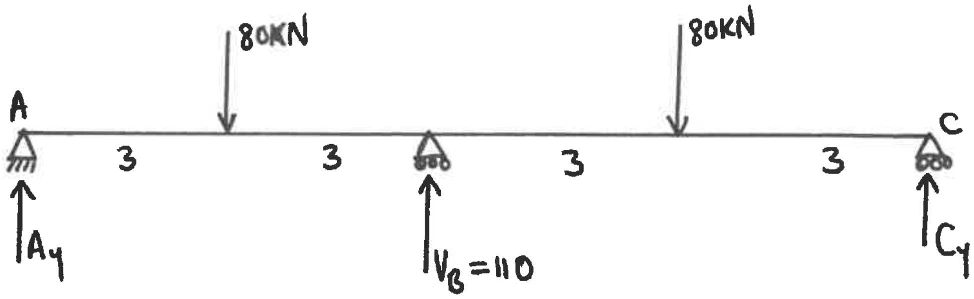
$$\Rightarrow X_1 = -\frac{\Delta_1}{\delta_{11}}$$

$$X_1 = -\left(-\frac{3960}{EI}\right) \div \left(\frac{36}{EI}\right)$$

$$X_1 = \frac{3960}{EI} \times \frac{EI}{36}$$

$$X_1 = 110\,kN$$

STATICS

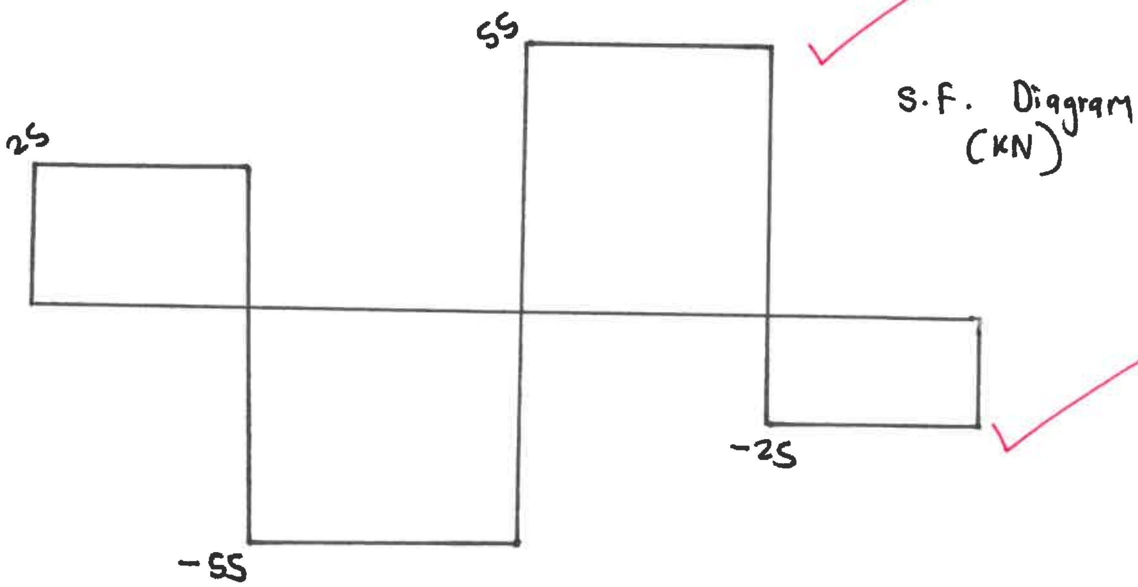


$\circlearrowright + \; \Sigma M_A = 0;$ $\quad 80(3) - 110(6) + 80(9) - C_y(12) = 0$

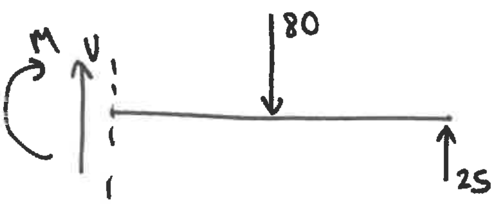
$C_y = 25\text{ kN}$

$\circlearrowright + \; \Sigma M_c = 0;$ $\quad -80(3) + 110(6) - 80(9) + A_y(12) = 0$

$A_y = 25\text{kN}$



Moment at centre :-

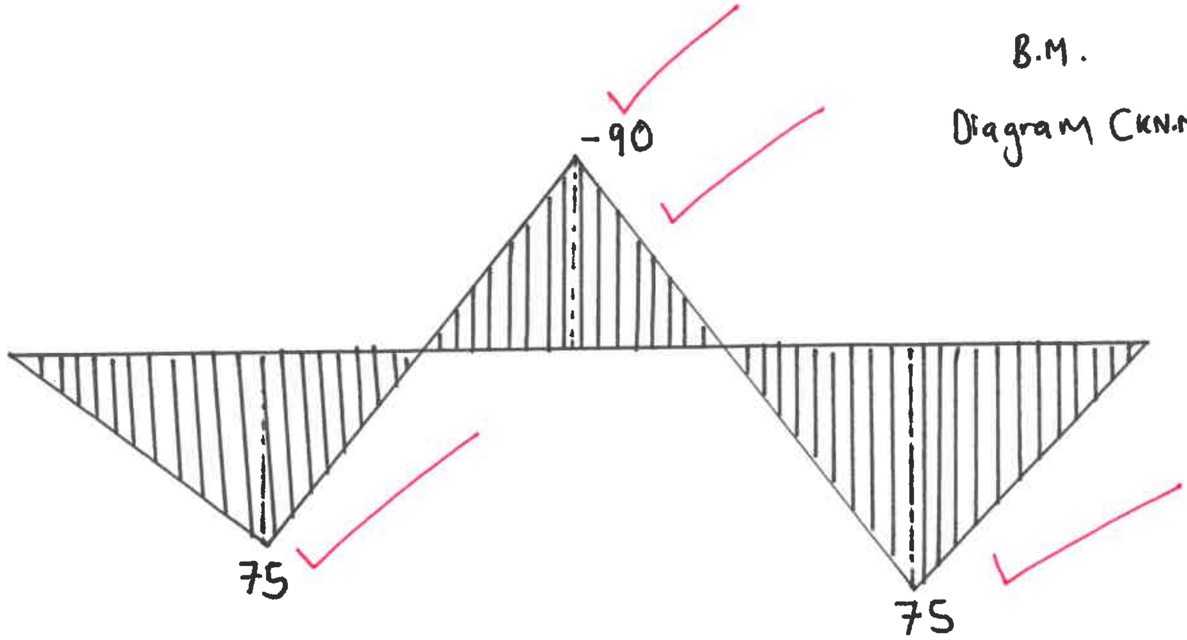


$\circlearrowright + \; \Sigma M = 0;$ $\quad M + 80(3) - 25(6) = 0$

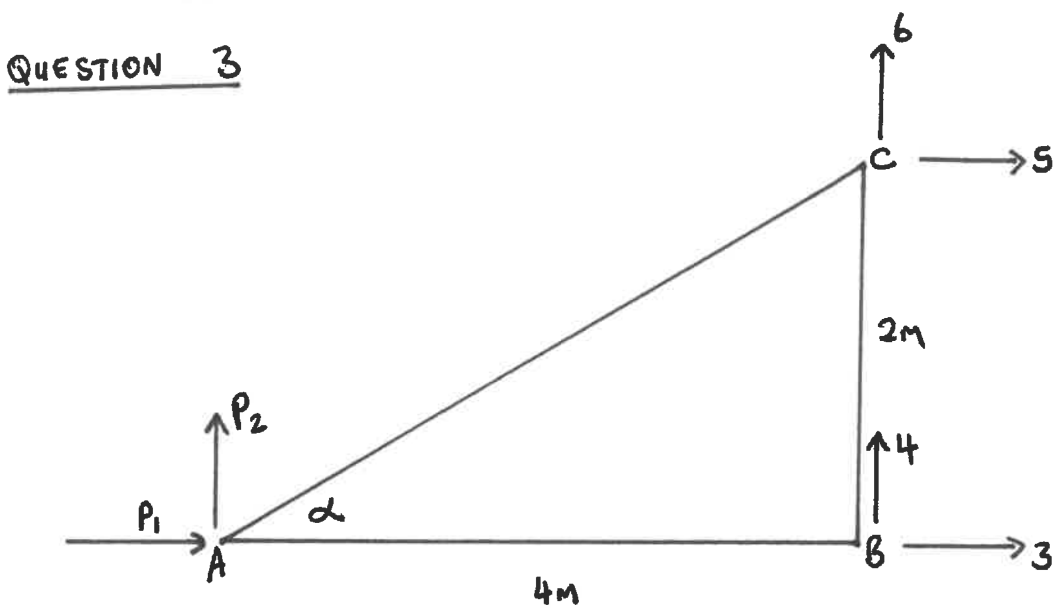
$M = -90\text{KN.M}$

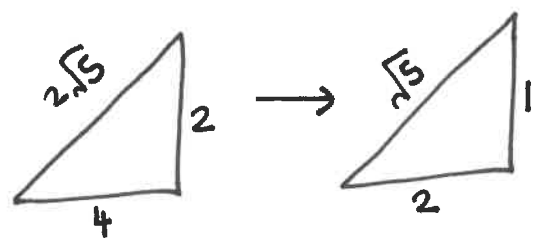
B.M.

Diagram (KN.M)



QUESTION 3





$E = 210\,GPa = 210 \times 10^3\,MPa = 210\,000\,N/mm^2$

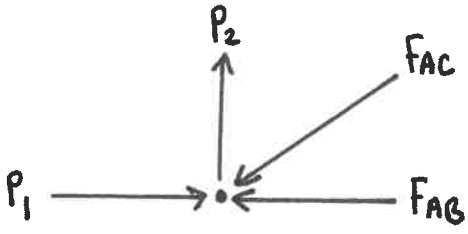
$A = 4000\,mm^2$

Let $X_1 = 1$

$$F_{AB} = \frac{EAx}{L} = \frac{(210\,000)(4000)(-1)}{4000} = -210\,000\,N$$

$$F_{AC} = \frac{EAx}{L} = \frac{(210\,000)(4000)\left(-\frac{2}{\sqrt{5}}\right)}{4472.14} = -167\,999.85\,N$$

Joint A :-



$\xrightarrow{+} \Sigma f_x = 0;\quad P_1 - F_{AB} - \frac{2}{\sqrt{5}} F_{AC} = 0$

$P_1 = F_{AB} + \frac{2}{\sqrt{5}} F_{AC}$

$P_1 = 210\,000 + \frac{2}{\sqrt{5}}(167\,999.85)$

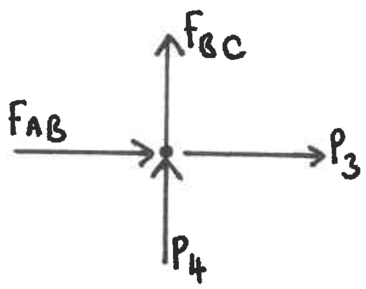
$P_1 = 360\,263.63\text{ N}$ ✓

$+\uparrow \Sigma f_y = 0;\quad P_2 - \frac{1}{\sqrt{5}} F_{AC} = 0$

$P_2 = \frac{1}{\sqrt{5}}(167\,999.85)$

$P_2 = 75\,131.82\text{ N}$ ✓

Joint B :

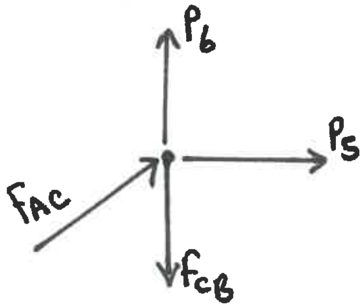


$\xrightarrow{+} \Sigma f_x = 0;\quad P_3 + F_{AB} = 0$

$P_3 = -210\,000\text{ N}$ ✓

$+\uparrow \Sigma f_y = 0;\quad P_4 = 0\text{ kN}$

Joint C :



$\xrightarrow{+} \Sigma f_x = 0;\quad P_5 + \frac{2}{\sqrt{5}} F_{AC} = 0$

$P_5 = -\frac{2}{\sqrt{5}}(167\,999.85)$

$P_5 = -150\,263.63\text{ N}$ ✓

$+\uparrow \Sigma F_y = 0;\quad P_6 + \frac{1}{\sqrt{5}} F_{AC} = 0$

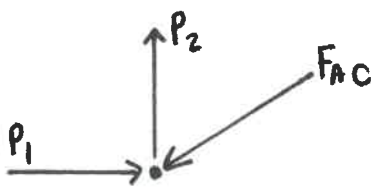
$P_6 = -\frac{1}{\sqrt{5}}(167\,999.85)$

$P_6 = -75\,131.82\text{ N}$ ✓

Let $X_2 = 1$

$$F_{AC} = \frac{EAx}{L} = \frac{(210\,000)(4000)\left(-\frac{1}{\sqrt{5}}\right)}{4472.14} = -83\,999.93 \text{ N}$$

Joint A :



$\xrightarrow{+} \Sigma F_x = 0;$ $P_1 - \frac{2}{\sqrt{5}} F_{AC} = 0$

$$P_1 = \frac{2}{\sqrt{5}}(83\,999.93)$$

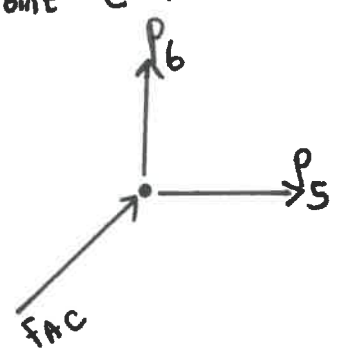
$$P_1 = 75\,131.82 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $P_2 - \frac{1}{\sqrt{5}} F_{AC} = 0$

$$P_2 = \frac{1}{\sqrt{5}}(83\,999.93)$$

$$P_2 = 37\,565.91 \text{ N}$$

Joint C :



$\xrightarrow{+} \Sigma F_x = 0;$ $P_5 + \frac{2}{\sqrt{5}} F_{AC} = 0$

$$P_5 = -\frac{2}{\sqrt{5}}(83\,999.93)$$

$$P_5 = -75\,131.82 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $P_6 + \frac{1}{\sqrt{5}} F_{AC} = 0$

$$P_6 = -\frac{1}{\sqrt{5}}(83\,999.93)$$

$$P_6 = -37\,565.91 \text{ N}$$

Joint B :-

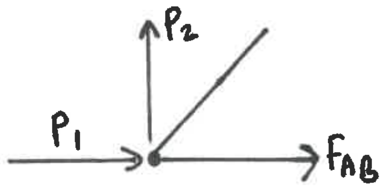
$P_3 = 0$ N $\&$

$P_4 = 0$ N

Let $X_3 = 1$

$$F_{AB} = \frac{EAx}{L} = \frac{(210\,000)(4000)(1)}{4000} = 210\,000\ \text{N}$$

Joint A:

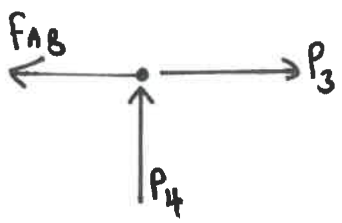


$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad P_1 + F_{AB} = 0$

$P_1 = -F_{AB} = -210\,000\text{N}$

$+\uparrow \Sigma F_y = 0;\quad P_2 = 0\text{N}$

Joint B:



$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad P_3 - F_{AB} = 0$

$P_3 = 210\,000\text{N}$

$P_4 = 0\text{N}$

Joint C:

clearly, $P_5 = 0\text{N}$ & $P_6 = 0\text{N}$

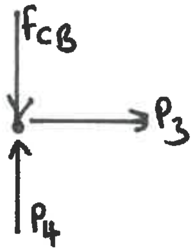
Let $X_4 = 1$

$$f_{BC} = \frac{EAx}{L} = \frac{(210\,000)(4000)(-1)}{2000} = -420\,000N$$

Joint A :

$P_1 = 0N$ & $P_2 = 0N$

Joint B :

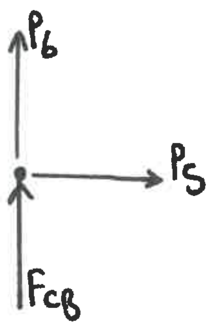


$+\uparrow \Sigma F_y = 0;\quad -F_{CB} + P_4 = 0$

$P_4 = +420\,000N$

$P_3 = 0N$

Joint C :



$+\uparrow \Sigma F_y = 0;\quad P_6 + F_{CB} = 0$

$F_{CB} = -P_6$

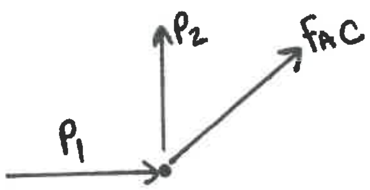
$P_6 = -420\,000N$

$P_5 = 0N$

Let $X_5 = 1$

$$F_{AC} = \frac{EAx}{L} = \frac{(210\,000)(4000)\left(\frac{2}{\sqrt{5}}\right)}{4472.14} = 167\,999.85 \text{ N}$$

Joint A :



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $P_1 + \frac{2}{\sqrt{5}} F_{AC} = 0$

$P_1 = -\frac{2}{\sqrt{5}}(167\,999.85)$

$P_1 = -150\,263.63$ N

$+\uparrow \Sigma F_y = 0;$ $P_2 + \frac{1}{\sqrt{5}} F_{AC} = 0$

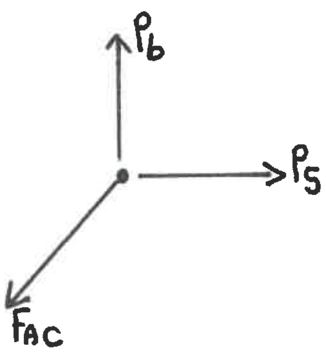
$P_2 = -\frac{1}{\sqrt{5}}(167\,999.85)$

$P_2 = -75\,131.82$

Joint B:

$P_3 = 0$N & $P_4 = 0$N

Joint C :



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $P_5 - \frac{2}{\sqrt{5}} F_{AC} = 0$

$P_5 = \frac{2}{\sqrt{5}}(167\,999.85)$

$P_5 = 150\,263.63$

$+\uparrow \Sigma F_y = 0;$

$P_6 - \frac{1}{\sqrt{5}} F_{AC} = 0$

$P_6 = \frac{1}{\sqrt{5}}(167\,999.85)$

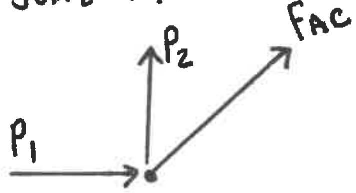
$P_6 = 75\,131.82$

Let $X_6 = 1$

$$F_{CB} = \frac{EAx}{L} = \frac{(210\,000)(4000)(1)}{2000} = 420\,000\text{ N}$$

$$F_{CA} = \frac{EAx}{L} = \frac{(210\,000)(4000)(\frac{1}{\sqrt{5}})}{4472.14} = 83999.92\text{ N}$$

Joint A :



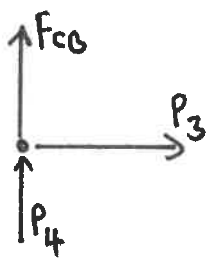
$\xrightarrow{+} \Sigma F_x = 0;\quad P_1 + \frac{2}{\sqrt{5}} F_{AC} = 0$

$P_1 = \frac{-2}{\sqrt{5}}(83\,999.92) = -75\,131.82\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad P_2 + \frac{1}{\sqrt{5}} F_{AC} = 0$

$P_2 = -\frac{1}{\sqrt{5}}(83\,999.92) = -37\,565.91\text{ N}$

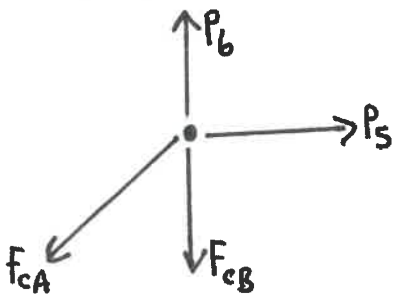
Joint B :



$P_3 = 0\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad P_4 + F_{CB} = 0;\quad P_4 = -420\,000\text{ N}$

Joint C :-



$\xrightarrow{+} \Sigma F_x = 0;\quad P_5 - \frac{2}{\sqrt{5}} F_{CA} = 0;\quad P_5 = \frac{2}{\sqrt{5}}(83\,999.92)$

$P_5 = 75\,131.82\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad P_6 - F_{CB} - \frac{1}{\sqrt{5}} F_{CA} = 0$

$P_6 = 420\,000 + \frac{1}{\sqrt{5}}(83\,999.92)$

$P_6 = 457565.91\text{ N}$

THE GLOBAL STIFFNESS MATRIX

P/X	1	2	3	4	5	6
1	360 263.63	75 131.82	-210 000	0	-150 263.63	-75 131.82
2	75 131.82	37 565.91	0	0	-75 131.82	-37 565.91
3	-120 000	0	210 000	0	0	0
4	0	0	0	420 000	0	-420 000
5	-150 263.63	-75 131.82	0	0	150 263.63	75 131.82
6	-75 131.82	-37 565.91	0	-420 000	75 131.82	457565.91