UNIVERSITY
OF
JOHANNESBURG

PROGRAM	:	BENGTECH *CIVIL ENGINEERING*
SUBJECT	:	**PRE-STRESSED CONCRETE DESIGN**
CODE	:	**PSCC1B3**
DATE	:	SUMMER EXAMINATION NOVEMBER 2019
DURATION	:	XXX
WEIGHT	:	40 : 60
TOTAL MARKS	:	100

EXAMINER	:	MR GD ROBERTS	Sanso Number
MODERATOR	:	XXX	File Number
NUMBER OF PAGES	:	3 PAGES	

INSTRUCTIONS	:	QUESTION PAPERS MUST BE HANDED IN.
REQUIREMENTS	:	NA.

INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1

a) What are the important losses of pre-stress? Give a short description of each. (5)

b) List the factors influencing deflections. (3)

c) Define pre-stressed concrete. State advantages as over reinforced concrete. (3)

d) Distinguish between pre-tensioning and post-tensioning. (3)

e) What are the stages to be considered in the design of pre-stressed concrete section under flexure? (2)

f) Define Bursting tension. (3)

g) Define partial pre-stressing. (3)

h) What do you understand the ultimate and serviceability limit states? (3)

QUESTION 2

A rectangular concrete beam of cross section 300 mm deep and 200 mm wide.

Is pre-stressed by means of 15 wires of 5 mm diameter located 65 mm from the bottom of the beam and 3 wires of diameter of 5 mm, 25 mm from the top.

Assuming the pre-stress in the steel as 840 MPa, calculate the stresses at the extreme fibres of the mid span section.

The beam is supporting its own weight over a span of 6m. If a uniformly distributed live load of 6 kN/m is imposed, evaluate the maximum working stress in concrete. The density of concrete is 24 kN/m^3.

(20)

QUESTION 3

A pre-stressed concrete beam of rectangular section 120 mm wide and 300 mm deep spans over 6 m.

$A_s = 150$ mm^2

The beam is pre-stressed by a straight cable carrying an effective force of 180 kN at an eccentricity of 50 mm.

If it supports an imposed load of 4 kN/m and the modules of elasticity of concrete is 38 GPa.

Compute the deflection at the following stages:

i) Upward deflection under (pre-stress + self weight) and

ii) Find downward deflation under (pre-stress + self weight + imposed load) inducing the effects of creep and shrinkage. Assume the creep coefficient to be 1.80.

(20)

QUESTION 4

A rectangular concrete beam of cross section 120 mm wide and 300 mm deep.

The beam is pre-stressed by a straight cable carrying an effective force of 180 kN at an eccentricity of 50 mm from the centre of the section.

The beam supports an imposed load of 3.14 kN/m over a span of 6 m. If the modulus of rupture of concrete is 5 MPa, evaluate the load factor against cracking assuming the selfweight of concrete as 24 kN/m^3.

(20)

QUESTION 5

A pre-tensioned beam of rectangular cross-section 150 mm wide and 300 mm deep.

The beam is pre-stressed by 8 x 7 mm diameter wires located 100 mm from the bottom of the beam. Beam Length = 10 m

If the wires are initially tensioned to a stress of 1100 N/mm^2, calculate the effective stress after all losses given the following:

- Relaxation of steel = 70 N/mm^2
- Shrinkage of concrete = 300x10^{-6}
- Creep of concrete = 1.6
- E_s = 210 GPa
- E_c = 31.5 GPa

(15)

4/...

Table 29 - Compressive stresses f_{cu} in concrete for serviceability limit states

1	2
Nature of loading	Allowable compressive stresses
Design load in bending	0,33 f_{cu} In continuous beams and other statically indeterminate structures, this may be increased to 0,4 f_{cu} within the range of support moments.
Design load in direct compression	0,25 f_{cu}

Table 30 - Flexural tensile stresses for class 2 elements (serviceability limit state (cracking))

1	2	3	4	5
Type of element	Limiting design stress*) MPa			
	Concrete grade			
	30	40	50	60
Pre-tensioned Post-tensioned	- 2,1	2,9 2,3	3,2 2,55	3,5 2,8

*) The limiting tensile stresses are $0{,}45\sqrt{f_{cu}}$ for pre-tensioned members and $0{,}36\sqrt{f_{cu}}$ for post-tensioned members.

$b := 200\ \text{mm}$ $\quad d := 300\ \text{mm}$ $\quad L := 6\ \text{m}$

$e_{top} := -(d - 25\ \text{mm}) = -275\ \text{mm}$ $\quad e_{bot} := (d - 65\ \text{mm}) = 235\ \text{mm}$ $\quad A := b \cdot d = 60000\ \text{mm}^2$

$A_{sbot} := 15 \cdot \dfrac{(5\ \text{mm})^2 \cdot \pi}{4} = 294.5243\ \text{mm}^2$ $\quad A_{stop} := 3 \cdot \dfrac{(5\ \text{mm})^2 \cdot \pi}{4} = 58.9049\ \text{mm}^2$

$P_{bot} := (-A_{sbot}) \cdot 840\ \text{MPa} = -247.4004\ \text{kN}$ $\quad P_{top} := (-A_{stop}) \cdot 840\ \text{MPa} = -49.4801\ \text{kN}$

$I := \dfrac{b \cdot d^3}{12} = 4.5 \cdot 10^8\ \text{mm}^4$ $\quad \gamma_c := 24\ \dfrac{\text{kN}}{\text{m}^3}$ $\quad w_d := A \cdot \gamma_c = 1.44\ \dfrac{\text{kN}}{\text{m}}$ $\quad w_L := 6\ \dfrac{\text{kN}}{\text{m}}$

$M_d := \dfrac{w_d \cdot L^2}{8} = 6.48\ \text{kN m}$ $\quad M_L := \dfrac{w_L \cdot L^2}{8} = 27\ \text{kN m}$ $\quad M_{Tot} := 1.2 \cdot M_d + 1.6 \cdot M_L$

$y_t := -\dfrac{d}{2}$ $\quad y_b := \dfrac{d}{2}$ $\quad z_t := \dfrac{I}{y_t} = -3 \cdot 10^6\ \text{mm}^3$ $\quad z_b := \dfrac{I}{y_b} = 3 \cdot 10^6\ \text{mm}^3$

Stress due to prestress:

$$\sigma_{Pstop} := \frac{P_{top}}{A} + \frac{P_{bot}}{A} + \frac{P_{top} \cdot e_{top}}{z_t} + \frac{P_{bot} \cdot e_{bot}}{z_t} = 9.896\ \text{MPa}$$

$$\sigma_{Psbot} := \frac{P_{top}}{A} + \frac{P_{bot}}{A} + \frac{P_{top} \cdot e_{top}}{z_b} + \frac{P_{bot} \cdot e_{bot}}{z_b} = -19.792\ \text{MPa}$$

At serviceability:

$\sigma_{Stop} := \sigma_{Pstop} + \dfrac{M_{Tot}}{z_t} = -7.096\ \text{MPa}$ $\quad \sigma_{Sbot} := \sigma_{Psbot} + \dfrac{M_{Tot}}{z_b} = -2.8\ \text{MPa}$

$b := 120\ \text{mm}$ $\quad d := 300\ \text{mm}$ $\quad I_c := \frac{b \cdot d^3}{12} = 2.7 \cdot 10^8\ \text{mm}^4$

$A_c := b \cdot d = 36000\ \text{mm}^2$ $\quad P := (-180)\ \text{kN}$ $\quad A_{ps} := 150\ \text{mm}^2$

$y_{top} := \left(-\frac{d}{2}\right)$ $\quad y_{bot} := \frac{d}{2}$ $\quad E_c := 38\ \text{GPa}$ $\quad A_s := 0$

$z_{top} := \frac{I_c}{y_{top}} = -1.8 \cdot 10^6\ \text{mm}^3$ $\quad z_{bot} := \frac{I_c}{y_{bot}} = 1.8 \cdot 10^6\ \text{mm}^3$

$e_p := 50\ \text{mm}$ $\quad L := 6\ \text{m}$ $\quad w_d := 24\ \frac{\text{kN}}{\text{m}^3} \cdot A_c = 0.864\ \frac{\text{kN}}{\text{m}}$ $\quad w_{ll} := 4\ \frac{\text{kN}}{\text{m}}$

$\phi_u := 1.8$

Prestress + self weight

Deflection due to prestress

$e_1 := e_p$ $\quad \kappa_{t1} := \frac{P \cdot e_1}{E_c \cdot I_c} = -8.7719 \cdot 10^{-7}\ \frac{\text{rad}}{\text{mm}}$

$$\delta_{tp} := \left(\frac{1}{8}\right) \cdot \kappa_{t1} \cdot L^2 = -3.9474\ \text{mm}$$

Deflection due to deadload

$$\delta_{td} := \frac{5 \cdot w_d \cdot L^4}{384 \cdot E_c \cdot I_c} = 1.4211\ \text{mm}$$

$$\delta_{tpd} := \delta_{tp} + \delta_{td} = -2.5263\ \text{mm}$$

Losses from creep and shrinkage

Rough Estimate used for further calculations

Concrete Shrinkage:

$$Thk_{eff} := \frac{2 \cdot b \cdot d}{(2 \cdot b + 2 \cdot d)} = 85.7143\ \text{mm}$$

$$\varepsilon_{sh} := 400 \cdot 10^{-6}\ \frac{\text{mm}}{\text{mm}}$$

Concrete Creep:

$$M_p := \frac{w_d \cdot L^2}{8} = 3.888 \text{ kN m}$$

$$f_{c\_cgs} := \frac{P}{A_c} + \frac{P \cdot \left(e_p^{\,2}\right)}{I_c} + \frac{M_p \cdot e_p}{I_c} = -5.9467 \text{ MPa}$$

$$\varepsilon_c := \frac{\phi_u}{E_c} \cdot \left(-f_{c\_cgs}\right) = 0.0003 \frac{\text{mm}}{\text{mm}}$$

$$\varepsilon_{loss} := \varepsilon_c + \varepsilon_{sh} = 0.0007 \frac{\text{mm}}{\text{mm}}$$

$$\sigma_{loss} := \varepsilon_{loss} \cdot E_c = 25.904 \text{ MPa}$$

$$\eta := \left(1 - \frac{\sigma_{loss}}{\left(-\frac{P}{A_{ps}}\right)}\right) = 0.9784$$

Pre-stress, Self-weight, Imposed load, creep and shrinkage

Deflection due to prestress (with Creep)

Midspan

$$\kappa_{LT1} := \frac{P \cdot e_1}{E_c \cdot I_c} \cdot \left(\eta + \left(\frac{1+\eta}{2}\right) \cdot \phi_u\right) = -2.4202 \cdot 10^{-6} \frac{\text{rad}}{\text{mm}}$$

$$\delta_{LTp} := \left(\frac{1}{8}\right) \cdot \kappa_{LT1} \cdot L^2 = -10.8907 \text{ mm}$$

Deflection due to permanent Loads

$$w_{perMin} := 1 \cdot \left(w_d\right) = 0.864 \frac{\text{kN}}{\text{m}}$$

$$w_{perMax} := 1.1 \cdot \left(w_d\right) = 0.9504 \frac{\text{kN}}{\text{m}}$$

Favourable

Un − Favourable

$$E_{eff} := \frac{E_c}{\left(1 + \phi_u\right)} = 13.5714 \text{ GPa}$$

$$\delta_{LTp1min} := \frac{5 \cdot w_{perMin} \cdot L^4}{384 \cdot E_{eff} \cdot I_c} = 3.9789 \text{ mm}$$

$$\delta_{LTp1max} := \frac{5 \cdot w_{perMax} \cdot L^4}{384 \cdot E_{eff} \cdot I_c} = 4.3768 \text{ mm}$$

Deflection due to shrinkage

midspan

$\rho := \frac{100 \cdot A_{ps}}{A_c} = 0.4167$

Note Ac is used instead of bd

$A'_s := 0 \text{ mm}^2$

$\rho' := \frac{100 \cdot A'_s}{A_c} = 0$

$K_{cs} := 0.7 \cdot \sqrt{\rho \cdot \left(1 - \frac{\rho'}{\rho}\right)} = 0.4518$

$\kappa_s := 0.125$

$\delta_{LTsh} := \kappa_s \cdot \left(K_{cs} \cdot \frac{\left|-400 \cdot 10^{-6}\right|}{\left(-y_{top} + y_{bot}\right)}\right) \cdot L^2 = 2.7111 \text{ mm}$

Deflection due to live load

$\delta_{LTll} := \frac{5 \cdot w_{ll} \cdot L^4}{384 \cdot E_c \cdot I_c} = 6.5789 \text{ mm}$

$\delta_{LTmin} := \delta_{LTp} + \delta_{LTplmin} + \delta_{LTsh} = -4.2007 \text{ mm}$

$\delta_{LTmax} := \delta_{LTp} + \delta_{LTplmax} + \delta_{LTsh} + \delta_{LTll} = 2.7761 \text{ mm}$

b) class 2 elements:

1) ensure that the tensile stresses do not exceed the flexural tensile stresses given in table 30;

Table 30 - Flexural tensile stresses for class 2 elements (serviceability limit state (cracking))

1	2	3	4	5
Type of element	Limiting design stress*) MPa			
	Concrete grade			
	30	40	50	60
Pre-tensioned	-	2,9	3,2	3,5
Post-tensioned	2,1	2,3	2,55	2,8

*)The limiting tensile stresses are $0{,}45\sqrt{f_{cu}}$ for pre-tensioned members and $0{,}36\sqrt{f_{cu}}$ for post-tensioned members.

2) the stress obtained from table 30 may be increased by up to 1,7 MPa, provided that it is shown by tests that such enhanced stress does not exceed three-quarters of the tensile stress calculated from the loading in the performance test corresponding to the appearance of the first crack; where such increase is used, ensure that the stress in the concrete due to prestress after losses will be at least 8,0 MPa; distribute pre-tensioned tendons well throughout the tension zone of the section and supplement post-tensioned tendons, if necessary, with unstressed reinforcement located near the tension face of the member;

3) where a service load is of a temporary nature and is exceptionally high in comparison with the load normally carried, a higher calculated tensile stress is allowable, provided that under normal service conditions the stress is compressive enough to ensure that any cracks that might have occurred, close up; ensure that this increase in stress will not exceed 1,0 MPa;

$F_{cu} := 45\ \text{MPa}$

$f_{limit} := 5\ \text{MPa}$

Rectangle

$b := 0.12\ \text{m}$ $d := 0.3\ \text{m}$

$Area := b \cdot d = 0.036\ \text{m}^2$

$y_{bot} := \frac{d}{2} = 0.15\ \text{m}$

$y_{top} := \frac{-d}{2} = -0.15\ \text{m}$

$I_{xx} := \frac{b \cdot d^3}{12} = 2.7 \cdot 10^8\ \text{mm}^4$

$e_t := 50\ \text{mm}$ $P_t := -180\ \text{kN}$

$Z_{bot} := \frac{I_{xx}}{y_{bot}} = 0.002\ \text{m}^3$ $Z_{top} := \frac{I_{xx}}{y_{top}} = -0.002\ \text{m}^3$

$$M_{cr} := f_{limit} \cdot Z_{bot} - P_t \cdot \left(\frac{Z_{bot}}{Area} + e_t \right) = 27\ \text{kN m}$$

$w_d := 24\ \frac{\text{kN}}{\text{m}^3} \cdot b \cdot d = 0.864\ \frac{\text{kN}}{\text{m}}$ $w_{LL} := 3.14\ \frac{\text{kN}}{\text{m}}$ $L := 6\ \text{m}$

$M_d := \frac{w_d \cdot L^2}{8} = 3.888\ \text{kN m}$ $M_{LL} := \frac{w_{LL} \cdot L^2}{8} = 14.13\ \text{kN m}$

$M_{app} := M_d + M_{LL} = 18.018\ \text{kN m}$

$FoS_T := \frac{M_{cr}}{M_d} = 6.9444$ $FoS_S := \frac{M_{cr}}{M_{app}} = 1.4985$

Exam 1: Question 5

$f_{pJ'} := 1100\ \text{MPa}$ $L_b := 10\ \text{m}$

$E_p := 210\ \text{GPa}$ $E_{ci} := 31.5\ \text{GPa}$

$b_{beam} := 150\ \text{mm}$ $d_{beam} := 300\ \text{mm}$ $A_c := b_{beam} \cdot d_{beam} = 45000\ \text{mm}^2$ $I_c := \dfrac{b_{beam} \cdot d_{beam}^{\ 3}}{12}$

$y_{top} := -\dfrac{d_{beam}}{2}$ $y_{bot} := \dfrac{d_{beam}}{2}$ $Z_{top} := \dfrac{I_c}{y_{top}}$ $Z_{bot} := \dfrac{I_c}{y_{bot}}$ $w_d := 24\ \dfrac{\text{kN}}{\text{m}^3} \cdot A_c$

$A_{ps} := \dfrac{8 \cdot (7\ \text{mm})^2 \cdot \pi}{4} = 307.8761\ \text{mm}^2$ $e_p := 100\ \text{mm}$

Elastic Shortening:

$P_{J'} := (-f_{pJ'}) \cdot A_{ps} = -338.6637\ \text{kN}$ $M_D := \dfrac{w_d \cdot L_b^{\ 2}}{8} = 13.5\ \text{kN m}$

$f_{c\_cgs\_PJ'} := \dfrac{P_{J'}}{A_c} + \dfrac{P_{J'} \cdot e_p^{\ 2}}{I_c} = -17.5603\ \text{MPa}$ $f_{c\_cgs\_D} := \dfrac{M_D \cdot e_p}{I_c} = 4\ \text{MPa}$

$\eta_{pt} := \dfrac{E_p}{E_{ci}} = 6.6667$ $\Delta f_{pES} := \dfrac{f_{c\_cgs\_PJ'} + f_{c\_cgs\_D}}{\dfrac{1}{\eta_{pt}} - \dfrac{f_{c\_cgs\_PJ'}}{f_{pJ'}}} = -81.7065\ \text{MPa}$

$f_{pt} := f_{pJ'} + \Delta f_{pES} = 1018.2935\ \text{MPa}$ and $P_t := -f_{pt} \cdot A_{ps} = -313.5082\ \text{kN}$

Steel Relaxation:

$\Delta f_{pR} := -70\ \text{MPa}$

Concrete Shrinkage:

$$\varepsilon_s := -300 \cdot 10^{-6}$$

$$\Delta f_{pS} := \varepsilon_s \cdot E_p = -63 \text{ MPa}$$

Concrete Creep:

$$w_p := 1 \cdot \left(w_d\right) = 1.08 \ \frac{\text{kN}}{\text{m}}$$

$$M_p := \frac{w_p \cdot L_b^{\ 2}}{8} = 13.5 \text{ kN m}$$

$$f_{c\_cgs} := \frac{P_t}{A_c} + \frac{P_t \cdot e_p^{\ 2}}{I_c} + \frac{M_p \cdot e_p}{I_c} = -12.256 \text{ MPa}$$

$$f_{c\_max} := \frac{P_t}{A_c} + \frac{P_t \cdot e_p}{Z_{bot}} + \frac{M_D}{Z_{bot}} = -14.9005 \text{ MPa}$$

$$\phi_{creep} := 1.6$$

$$\varepsilon_c := \frac{f_{c\_cgs}}{E_{ci}} \cdot \phi_{creep} = -0.0006$$

$$\Delta f_{pC} := \varepsilon_c \cdot E_p = -130.7305 \text{ MPa}$$

Check assumption of shrinkage and creep > 500 microstrain:

$$\left(\varepsilon_s + \varepsilon_c\right) \cdot 1000000 = -922.526$$

Therefore it is valid

Total loss:

$$\Delta f_{pT} := \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pS} + \Delta f_{pC} = -345.437 \text{ MPa}$$

$$f_{se} := f_{pJ'} + \Delta f_{pT} = 754.563 \text{ MPa}$$

$$\eta := \frac{f_{se}}{f_{pt}} = 0.741$$