Question 3

MEMO

UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE *CIVIL ENGINEERING*
SUBJECT	:	**Hydraulic Engineering 3A**
CODE	:	**HMG3A11**
DATE	:	SSA EXAMINATION 16 JULY 2019
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TOTAL MARKS	:	100

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MODERATOR	:	DR S NYENDE-BYAKIKA FILE NO: HMG3A 2019
NUMBER OF PAGES	:	4 PAGES AND 1 ANNEXURES

INSTRUCTIONS	:	QUESTION PAPERS MUST BE HANDED IN
REQUIREMENTS	:	3 SHEETS OF PAPER

INSTRUCTIONS TO STUDENTS

- PLEASE ANSWER ALL QUESTIONS
- PROVIDE SHORT AND PRECISE ANSWERS FOR THE THEORETICAL PART
- SHOW ALL THE STEPS FOR CALCULATIONS CLEARLY

QUESTION 1: THEORY [30 Marks]

1.1. Describe the occurrences of cavitation phenomenon in pump systems and the main detection techniques. (8)

- **Occurrences of Cavitation Phenomenon:**
    - a problem associated with a high elevation difference between pump and the fluid surface.
    - **Cavitation occurs**, for a piping system that include a pump, when the **absolute pressure at the pump inlet falls bellow the vapor pressure of the water**. It may occur at the **inlet of the pump** and on the **impeller blades**, particularly if the pump is mounted above the level in the suction reservoir. Under cavitation condition, the **water starts to vaporise** to form **small cavities of water vapour.** The **bubble collapse is audible** and it **sounds like pumping marbles and stones** or as if gravel is forced through the pump.

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- **Cavitation detection mechanisms**
    - optically by using a transparent pump volute, illuminated with a stroboscope.
    - closely listening to the noise made by the pump
    - check the volumetric flow rate.

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1.2. Discuss the occurrence of surges in water supply systems and state the potential mitigation mechanisms in big cities (7)

- Surges or hydraulic transients occur when the total surge pressure exceeds the static pressure in the system. Its occurrence is by the presence of air pockets within the supply system.

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- Surge control mechanisms:
    1. Surge tanks/chambers or relieve valves
    2. Check/air valves or vessels
    3. Fly wheels
    4. Slow valve closure
    5. Controlled opening and closure of pumps

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1.3. Explain the information that can be obtained from a typical pump curve. (5)

- **discharge on the x-axis,**
- **head on the left y-axis,**
- **pump power input on the right y-axis,**
- **pump efficiency as a percentage,**
- **speed of the pump (rpm).**
- **$NPSH_R$ of the pump**

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1.4. Discuss why the analysis of pipe network is complex? Also briefly discuss about the characteristics and application of Hardy-Cross Method. (10)

- **Pipe network analysis is complex:** 5
    - Because of the **uncertainties** about flow rate and its direction.
        - We do not necessarily know the direction of water flow. If one route is blocked, there are other routes for water to reach the consumer. We can't know the direction of water flow by eye inspection

- pipes are deliberately linked in a grid pattern to provide some redundancy. One pipe has to be taken out of commission for maintenance or repair, there are some other routes for the water to reach the majority of the consumers.

- **Hardy cross method:** $\boxed{5}$
  - Hardy-Cross method is an iterative procedure for estimating flow and headloss within a pipe network systems.
  - The method is only applicable to a closed loop system only.
  - Initially the flow will be assumed and correction will be made after each iteration using the closure error.
  - The Method is based on two basic hydraulic principles/Criteria:
    - Conservation of Mass Criteria: → At each node/junction, the total inflow must be equal to the total outflow
    - Conservation of Energy Criteria: → Head Loss Balance within the loop:

QUESTION 2 [16 Marks]

2.1 Derive Hazen Williams head loss Formula from the basic velocity Equation. (7)

$v = BCR^{0.63}S^{0.54}$

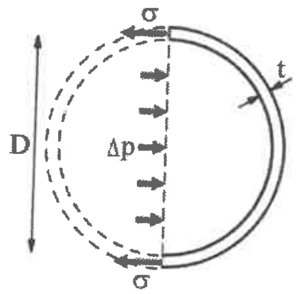
$$v = BCR^{0.63}S^{0.54}$$

$$S_o = \left(\frac{v}{BCR^{0.63}}\right)^{1/0.54} = \left(\frac{4Q/\pi D^2}{BC(D/4)^{0.63}}\right)^{1/0.54} = \left(\frac{4*4^{0.63}}{\pi}\right)^{1.852}\left(\frac{Q}{BCD^2D^{0.63}}\right)^{1.852} = \frac{7.89}{B^{1.85}}\frac{1}{C^{1.85}}\frac{1}{D^{4.87}}Q^{1.852} \quad \boxed{4}$$

$$S_o = \frac{h_L}{L} \Rightarrow h_L = S_o.L = \left(\frac{7.89}{B^{1.85}}\frac{1}{C^{1.85}}\frac{1}{D^{4.87}}Q^{1.852}\right) L \quad \boxed{1}$$

$$h_L = \frac{7.89}{B^{1.85}}\frac{L}{C^{1.85}}\frac{Q^{1.85}}{D^{4.87}} \quad \boxed{2}$$

2.2 Derive wave celerity and transient pressure equations for an elastic pipe carrying an incompressible fluid. (9)



From the diagram, an increase in pressure **ΔP** induces a hoop **stress** **σ**. If **D** is the **internal diameter of the pipe** and **t is the wall thickness.**

Equating forces per unit length:

$$2\sigma t = \Delta P.D$$

stress = Young's modulus * strain

$$\sigma = E * \frac{\pi.\Delta D}{\pi.D} = E\frac{\Delta D}{D}$$

$$\Rightarrow \frac{\Delta D}{D} = \frac{D}{2Et}\Delta P$$

This is the fractional change in diameter

Fractional change in area:

➢ From the geometry: $\Delta A \approx \frac{dA}{dD}\Delta D = \frac{2\pi D}{4}\Delta D \quad \Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta D}{D} = \frac{D}{Et}\Delta P$

The pressure change across the shock is the same as that of rigid pipes, still given by:

$$\Delta P = \rho c u_o$$

➢ **Continuity principle:** $\rho(c+u_o)A = (\rho+\Delta\rho)c(A+\Delta A)$

Divide by ρcA

$$1+\frac{u_o}{c} = \left(1+\frac{\Delta\rho}{\rho}\right)\left(1+\frac{\Delta A}{A}\right) = 1+\frac{\Delta\rho}{\rho}+\frac{\Delta A}{A}+2^{nd}\,d-order\ term$$

$$\Rightarrow \frac{u}{c} = \frac{\Delta\rho}{\rho}+\frac{\Delta A}{A}$$

- **From momentum:** $\frac{u}{c} = \frac{\Delta P}{\rho c^2}$
- **From compressibility:** $\frac{\Delta\rho}{\rho} = \frac{\Delta P}{K}$
- **From elasticity:** $\frac{\Delta A}{A} = \frac{D}{Et}\Delta P$

$$\Rightarrow \frac{1}{\rho c^2} = \frac{1}{K}+\frac{D}{Et}$$

➢ **Wave celerity for non-rigid pipes after substitution:**

$$c = \sqrt{\frac{1}{\rho\left[\frac{1}{K}+\frac{D}{Et}\right]}}$$

$$c = \sqrt{\frac{K'}{\rho}}$$

$$\frac{1}{K'} = \frac{1}{K}+\frac{D}{Et}$$

QUESTION 3 [17 Marks]

A uniform pipeline (500 m length, 200 mm diameter and roughness of 0.03 mm) conveys water at 18 °C between two reservoirs. The difference in water level between the two is maintained as 50 m. There are 5 valves (Gate valve, half closed) used in the pipe system located at some distance upstream of the second reservoir.

(a) Determine the steady discharge between the two reservoirs using:

a. Colebrook-white Equation (12)

~~b. Moody Diagram (6)~~

(b) Draw HGL and EGL between the two reservoirs (5)

Answer

$$H = h_f + \sum h_m$$

$$\sum h_m = 0.5\frac{v^2}{2g} + 1.0\frac{v^2}{2g} + 5*2.1\frac{v^2}{2g}$$

$$H = f\frac{L}{D}\frac{v^2}{2g} + 0.5\frac{v^2}{2g} + 1.0\frac{v^2}{2g} + 10.5\frac{v^2}{2g} \quad \boxed{1}$$

$$50 = f\left(\frac{500}{0.2}\right)\frac{v^2}{2g} + 12\frac{v^2}{2g} = 2500f\frac{v^2}{2g} + 12\frac{v^2}{2g} = 127.42fv^2 + 0.612v^2 \cdots\cdots(1)$$

$$50 = (127.42f + 0.612)v^2 \cdots\cdots\cdots(2)$$

$$v = \left[\frac{50}{127.42f + 0.612}\right]^{0.5} \cdots\cdots\cdots(3)$$

$$f = \frac{\left(\frac{50}{v^2}\right) - 0.612}{127.42} = \frac{0.3942}{v^2} - 0.0048 \cdots\cdots\cdots(4) \quad \boxed{2}$$

Temp = 18 °C→ρ = 998.6, μ =1.06*10^{-3} $\boxed{1}$

a) Colebrook-White Eqn

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.03}{3.7*200} + \frac{2.51}{R_e\sqrt{f}}\right)$$

$$R_e = \frac{\rho v D}{\mu} = \frac{998.6v*0.2}{1.06*10^{-3}} = 1.89*10^5 v \cdots\cdots\cdots(5) \quad \boxed{1}$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.03}{3.7*200} + \frac{2.51}{1.89*10^5 v\sqrt{f}}\right) \cdots\cdots\cdots(6) \quad \boxed{1}$$

Equation (1) and (5) can be solved simultaneously or Eq. (3) or (4) can be substituted in eq. (6) and solved by trial and error procedure.

Substituting Equ. (3) in Equ (6) and simplifying yields:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.03}{3.7*200} + \frac{2.51}{1.89*10^{5}\left[\frac{50}{127.42f+0.612}\right]^{0.5}\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = -2\log\left(4.05*10^{-5} + \frac{1.33*10^{-5}}{\left[\frac{50}{127.42f+0.612}\right]^{0.5}\sqrt{f}}\right) \cdots\cdots\cdots(7)$$

Solving Equ. (7) by trial and error procedure:

$f_{assumed}$	$f_{calculated}$	V	Q
0.02	0.0142	3.98	
0.0142	0.0143	4.54	
0.0143	0.0143	4.54	0.142

f	V	Re			f	Q
0.0200	3.978	751754	6.41498E-05	8.3856	0.0142	0.124895
0.0142	4.542	858374	6.50612E-05	8.3734	0.0143	0.142608
0.0143	**4.537**	**857436**	**6.50521E-05**	**8.3735**	**0.0143**	**0.142452**

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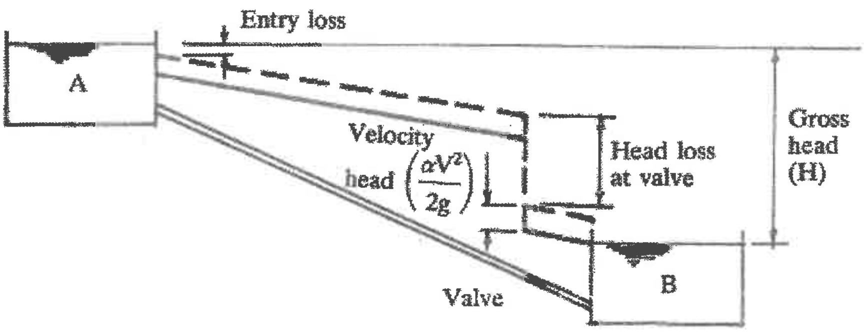
~~b) Using Moody Diagram~~

~~This involves the determination of f from Moody Diagram.~~

~~The solution is still iterative since the value of Re is not known.~~ ~~ε/D = 0.003/200 =0.00015~~

f	V	Re	ε/D	f	V	hf	hm	Q
0.0200	3.978	751754	0.00015	0.0143	1.183	2.54842	0.8554	0.0371
0.0143	4.532	856598	0.00015	**0.0150**	**1.155**	**2.54842**	**0.8155**	**0.0363**
0.0150	4.451	841322	0.00015	**0.0150**	**1.155**	**2.54842**	**0.8155**	**0.0363**
0.0200	3.978	751754	0.00015	0.0143	1.183	2.54842	0.8554	0.0371
0.0143	4.532	856598	0.00015	**0.0150**	**1.155**	**2.54842**	**0.8155**	**0.0363**
0.0150	4.451	841322	0.00015	**0.0150**	**1.155**	**2.54842**	**0.8155**	**0.0363**

c) Sketch HGL and EGL



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QUESTION 4 [20 Marks]

Three reservoirs connected at a single junction (Fig 3). The length (L), diameter (D), friction factor f and minor loss coefficient (K) of each pipe are given in Table below.

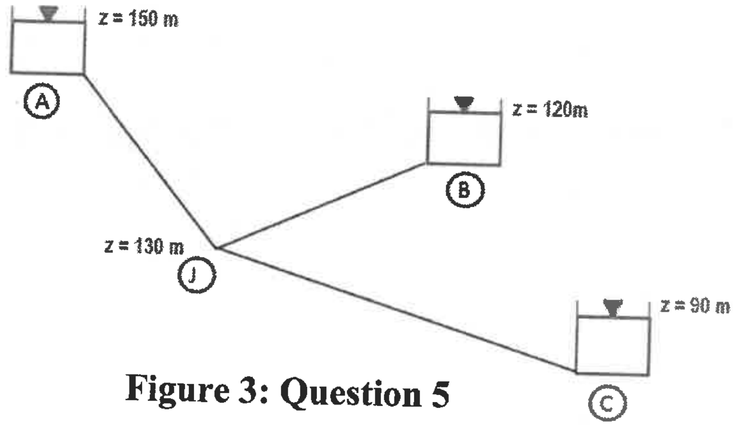


Figure 3: Question 5

	JA	JB	JC
L (m)	1600	1600	2400
D (m)	0.30	0.20	0.25
f	0.015	0.015	0.025
K	40	25	50

Calculate:

a) the flow in each pipe (12 Marks)

b) The reading of a pressure gauge attached to the junction J. (6 Marks)
[Hint: Start assuming pressure head at J = 120m]

Solution

$$h_L = f\frac{L}{D}\frac{v^2}{2g} + K\frac{v^2}{2g} \quad = \left(f\frac{L}{D} + K\right)\left(\frac{8}{\pi^2 g D^4}\right)Q^2$$

Pipe	H	L	D	f	K	hf/Q2		120	130	131.8
A	150	1600	0.3	0.015	40	1224	1224	30	20	18.2
B	120	1600	0.2	0.015	25	7489	7488	0	10	11.8
C	90	2400	0.25	0.025	50	6135	6134	30	40	41.8

Hj	Q_{JA}	Q_{JB}	Q_{JC}	Qnet outflow
120	-0.1565	0	0.06993	-0.08661
130	-0.1278	0.03654	0.08075	-0.01053
131.8	**-0.1219**	**0.03969**	**0.08254**	0.00031

Hj range of acceptance = 131-132 m

Q_{JA} **0.1226 m^3/s**

Q_{JB} **0.0394 m^3/s**

Q_{JC} **0.0824 m^3/s**

Pressure gauge readings:

$H_J = P/\rho g + z$

Gauge pressure, P_g = ρg (H_J-z) = 1000*9.81(131.6-130)= 15696 Pa

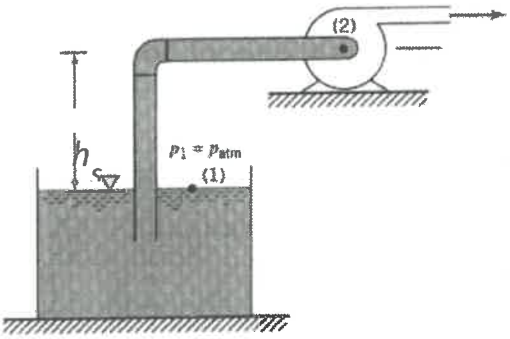
Absolute pressure, P_a = P_{atm} + P_{gauge} = 101 000 Pa + 15 696 Pa = **116 696 Pa = 116.7 KPa**

QUESTION 5 [17 Marks]

A pump was designed to satisfy the following system:

Q (m^3/hr)	4	7	10	15
h_L (m)	16	25	40	70

Check whether the pump is suitable or not using the given data and pump curve.



Given Data:

- Gauge Pressure = 10.5 KPa
- Vapour pressure head = 0.3 m
- Pipe diameter = 60 mm
- Suction depth = 5 m, delivery depth = 10 m

$$(\text{suction Part})\ h_L = \frac{32 \times V^2}{2g}$$

Solution

Draw the system curve and check the operation point

$H_{stat} = h_d + h_s = 10+5 = 15$ m

Q (m^3/hr)	4	7	10	15
h_L (m)	16	25	40	70
H_{system}	**31**	**40**	**55**	**85**

At operating point:
Q = 10 m^3/hr
H = 56 m
$NPSH_R$ = 5.0 m

P = (10.5+101.3)*10^3/(1000x9.81) **11.40 m**

Then Check $NPSH_A$

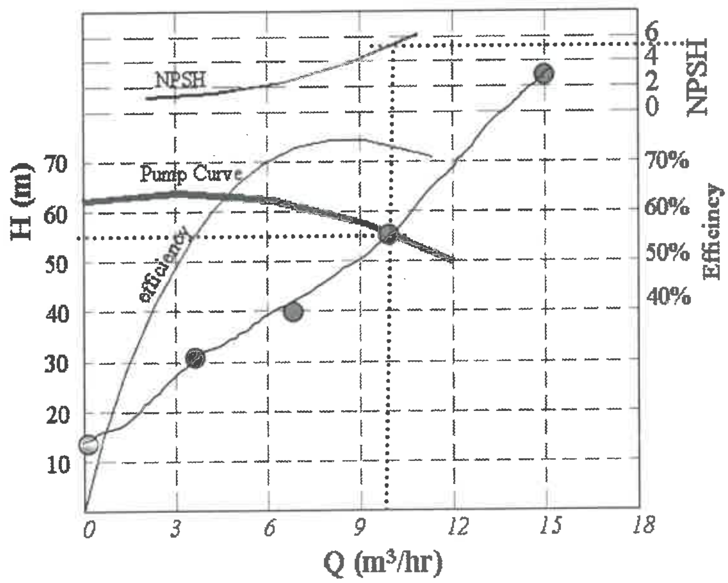


Figure. Typical pump curve

$$V = \frac{Q}{A} = \frac{10/3600}{\frac{\pi}{4} \times (0.06)^2} = 0.983 m/s$$

$$h_L = \frac{32 \times (0.983)^2}{2*9.81} = 1.6m$$

$$(\text{NPSH})_A = \pm h_S - h_{fS} - \sum h_{mS} + \frac{P_{atm}}{\gamma_{air}} - \frac{P_{Vapor}}{\gamma_{Vapor}}$$

$(NPSH)_A = -5-1.6+11.4-0.3$
$(NPSH)_A = 4.4 < 5.0$
$(NPSH)_A < (NPSH)_R$
The pump is not suitable.
Cavitation will occur

Formula Sheet Hydraulic Engineering 3A

$$z_1 + h_1 + \frac{V_1^2}{2g} + h_p = z_2 + h_2 + \frac{V_2^2}{2g} + h_f + h_s + h_t$$

$$Re = \frac{\rho V D}{\mu}$$

$$y_v = 14 \times \frac{D}{Re\sqrt{f}}$$

$$z_1 + h_1 + \frac{V_1^2}{2g} = z_2 + h_2 + \frac{V_2^2}{2g}$$

$$y_t = 184 \times \frac{D}{Re\sqrt{f}}$$

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{Re\sqrt{f}}{2.51}\right)$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$$

$$\frac{V_p}{V_{max}} = \left(\frac{2y}{D}\right)^{1/n}$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11} + \frac{6.9}{Re}\right]$$

$$\alpha = 1 + 2.7f$$

$$h_f = \frac{fLV^2}{2gD}$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right)$$

$$D_h = \frac{4A}{P}$$

$$\left[\frac{fL}{D^5}\right]_e = \sum_{i=1}^{N}\left[\frac{fL}{D^5}\right]_i$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{5.129}{Re^{0.89}}\right)$$

$$h_s = K\left(\frac{V^2}{2g}\right)$$

$$L_{eq} = \frac{KD}{f}$$

$$Q_1 = 0.5Q_0 + 0.5A(H_{begin} - H_{end})$$

$$A = \frac{1}{KQ_0}$$

$$\left[\sqrt{\frac{D^5}{fL}}\right]_e = \sum_{i=1}^{N}\left[\sqrt{\frac{D^5}{fL}}\right]_i$$

$$NPSHA = H_{abs} + H_s - H_f - H_{sec} - H_{vap}$$

$$P = \frac{\rho g v H}{t} = \rho g Q H$$

$$P_a = \frac{\rho g Q H}{\eta}$$

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}.\frac{N_1}{N_2}\right)$$

$$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}.\frac{N_1}{N_2}\right)^2$$

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}.\frac{N_1}{N_2}\right)^3$$

$$\frac{NPSHR_1}{NPSHR_2} = \left(\frac{N_1}{N_2}\right)^2$$

$$P = A\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$c = \sqrt{\frac{K}{\rho}}$$

$$PS = \frac{54EI}{D^3} \geq \frac{98950}{D}$$

$$\Delta h = \frac{\Delta p}{\rho g} = \frac{cV_0}{g}$$

$$\Delta h = \frac{\Delta p}{\rho g} = \frac{L}{g}\left(\frac{V_0}{\Delta t}\right)$$

$$\frac{\Delta Y}{D} = 100\left(\frac{D_L K P}{0.149PS + 0.061E'}\right)$$

$$c_e = \sqrt{\frac{1}{\rho\left(\frac{1}{K} + \frac{C_1 D}{TE}\right)}}$$

$$T = \frac{pD}{2fGJ}$$

$$C_D = \frac{1 - \exp\left[-2K\tan\theta\left(\frac{H}{B}\right)\right]}{2K\tan\theta}$$

$$W = C_D \rho g B^2$$

$$W = C_C \rho g D^2$$

$$C_C = \frac{\exp\left[2K\tan\varnothing\left(\frac{H}{D}\right)\right] - 1}{2K\tan\varnothing}$$

$$w = \frac{P(1 + IM)}{A}$$

$$W_{total} = wLS_L$$

$$\sigma = \rho g h\frac{1 + \sin\phi}{1 - \sin\phi} + 2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}$$

$$F = 2(pA + \rho QV)\sin\left(\frac{\theta}{2}\right)$$

$$L_e = L + 1.31D$$

$$Q = \frac{ND\sqrt{P}}{70}$$