FACULTY OF SCIENCE

ACADEMY OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

MODULE	**GRAPHICS** IT18X77
CAMPUS	AUCKLAND PARK CAMPUS (APK)
FSAO SSA	JULY - **MEMO**

DATE: 2020-07	**SESSION:** Morning
ASSESSOR(S):	**MR. A. MAGANLAL**
MODERATOR:	**DR. D VOGTS (NMU)**
DURATION: 120 MINUTES	**MARKS:** 100

Please read the following instructions carefully:

1. Answer **all** the questions.
2. Answer questions in order.
3. Answer only in the answer sheets provided.
4. Use diagrams where necessary to assist in your explanations.
5. Non-programmable calculators are allowed.
6. Round final answers to three decimal places.
7. Write *cleanly* and *legibly*.
8. This paper contains **6** question(s).
9. This paper consists of **7** page(s).

QUESTION 1: Rotation Calculation

Answer only one part. Either **Matrix** or **Quaternion**

(a) **Matrix**

Construct a matrix to rotate 347° (clockwise) around the axis specified by

$$(-11, 6, 22) \rightarrow (-3, 0, 17)$$

Rotate the point (22, 0, 3) around this axis.

Solution:

The direction vector around which rotation occurs is $(-3, 0, 17) - (-11, 6, 22) = (8, -6, -5)$. Normalizing this vector gives $\mathbf{u} = (0.715542, -0.536656, -0.447214)$✓✓. We need to translate the point so that $(-11, 6, 22)$ is transformed to the origin, rotate and then translate back again. Our final matrix is

$$M = T(\mathbf{o})R_{\mathbf{u}}(347^\circ)T(-\mathbf{o})✓✓$$

where $\mathbf{o} = (-11, 6, 22)$. The matrices are as follows:

$$T(\mathbf{o}) = \begin{pmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 22 \\ 0 & 0 & 0 & 1 \end{pmatrix} ✓ \qquad T(-\mathbf{o}) = \begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -22 \\ 0 & 0 & 0 & 1 \end{pmatrix} ✓$$

$$R_{\mathbf{u}}(347^\circ) = \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & 0 \\ 0.0907593 & 0.981752 & 0.167113 & 0 \\ -0.128923 & -0.154811 & 0.979496 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ✓$$

We now simply multiply to get M:

$$M = T(\mathbf{o})R_{\mathbf{u}}(347^\circ)T(-\mathbf{o})$$

$$T(\mathbf{o})R_{\mathbf{u}}(347^\circ) = \begin{pmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 22 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & 0 \\ 0.0907593 & 0.981752 & 0.167113 & 0 \\ -0.128923 & -0.154811 & 0.979496 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & -11 \\ 0.0907593 & 0.981752 & 0.167113 & 6 \\ -0.128923 & -0.154811 & 0.979496 & 22 \\ 0 & 0 & 0 & 1 \end{pmatrix} ✓✓$$

$$M = \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & -11 \\ 0.0907593 & 0.981752 & 0.167113 & 6 \\ -0.128923 & -0.154811 & 0.979496 & 22 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -22 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & -1.95036 \\ 0.0907593 & 0.981752 & 0.167113 & -2.56864 \\ -0.128923 & -0.154811 & 0.979496 & -0.038202 \\ 0 & 0 & 0 & 1 \end{pmatrix} ✓✓$$

We now rotate $\mathbf{p} = (22, 0, 3)$ around this axis simply by multiplying by M:

$$\mathbf{p}' = M\mathbf{p}\checkmark$$

$$= \begin{pmatrix} 0.987493 & -0.110443 & 0.11252 & -1.95036 \\ 0.0907593 & 0.981752 & 0.167113 & -2.56864 \\ -0.128923 & -0.154811 & 0.979496 & -0.038202 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 22 \\ 0 \\ 3 \\ 1 \end{pmatrix} \quad = \begin{pmatrix} 20.112 \\ -0.0706012 \\ 0.0639805 \\ 1 \end{pmatrix}$$

Thus we obtain the point $(20.112, -0.0706012, 0.0639805)\checkmark\checkmark\checkmark$.

(b) **Quaternion**
Construct a quaternion to rotate 347° (clockwise) about the axis (8, –6, –5).
Use the quaternion to rotate (33, –6, –19) around this axis.

Solution:

Let $\mathbf{p} = (33, -6, -19, 1) = (\mathbf{p}_v, p_w)$. To rotate 347° around (8, –6, –5) we create the quaternion $\mathbf{q} = (\mathbf{q}_v, q_w) = (\sin(173.5^\circ)\mathbf{u}, \cos(173.5^\circ))\checkmark$. The normalized axis $\mathbf{u} = (0.715542, -0.536656, -0.447214)\checkmark\checkmark$ gives $\mathbf{q} = (0.0810016i - 0.0607512j - 0.050626k - 0.993572)\checkmark$.
This gives $\mathbf{q}^* = (-0.0810016i + 0.0607512j + 0.050626k - 0.993572)\checkmark$. To perform the rotation we calculate:

$$\begin{aligned}
\mathbf{qpq}^*\checkmark\checkmark &= (0.0810016i - 0.0607512j - 0.050626k - 0.993572)(33i - 6j - 19k + 1)\mathbf{q}^* \\
&= ((-32.7879i - 1.67066j + 2.00479k - 2.67305)+ \\
&\quad (-0.303756i + 5.96143j - 0.48601k - 0.364507)+ \\
&\quad (1.15427i + 1.53903j + 18.8779k - 0.961894)+ \\
&\quad (0.0810016i - 0.0607512j - 0.050626k - 0.993572))\mathbf{q}^*\checkmark\checkmark \\
&= (-31.8564i + 5.76905j + 20.346k - 4.99303)\mathbf{q}^* \\
&= ((0.404443i - 1.64806j + 0.467303k - 2.58042)+ \\
&\quad (-1.23605i - 0.303332j - 1.93531k - 0.350477)+ \\
&\quad (0.292064i + 1.61276j - 0.252777k - 1.03004)+ \\
&\quad (31.6516i - 5.73197j - 20.2152k + 4.96093))\checkmark\checkmark\checkmark \\
&= (31.112i - 6.0706j - 21.936k + 1)
\end{aligned}$$

Thus we obtain the point $(31.112, -6.0706, -21.936)\checkmark\checkmark\checkmark$.

Total: 15

QUESTION 2: Lighting Equation

Write down the Phong lighting equation for a single colour light source and object (black and white model). Now calculate the viewed intensity of a point on an object given the following attributes:

- The object is not emissive at all.
- The ambient light intensity is 0.214.
- The object has an ambient coefficient of 0.936.
- The object has a diffuse reflection coefficient of 0.745.
- The object has a specular reflection coefficient of 0.996.
- The shininess (specular highlight) factor is 3.
- The intensity of the incoming light (both specular and diffuse) is 0.401.
- The point we are considering is (4, –11, –6).
- The normal at the surface is (–0.624242, 0.780303, 0.0380749).
- The light is positioned at (1, –8, 12).
- The viewer is positioned at (–6, 2, 16).

Solution:

$$I = I_e + \rho_a I_a^{in} + \rho_d I_d^{in}(\mathbf{l} \cdot \mathbf{n}) + \rho_s I_s^{in}(\mathbf{v} \cdot \mathbf{r})^f \checkmark\checkmark$$

We note that

- $I_e = 0 \checkmark \frac{1}{2}$
- $I_a = 0.214 \times 0.936 = 0.200304 \checkmark$
- $I_d^{in} = I_s^{in} = 0.401 \checkmark \frac{1}{2} \checkmark \frac{1}{2}$
- $\rho_d = 0.745 \checkmark \frac{1}{2}$
- $\rho_s = 0.996 \checkmark \frac{1}{2}$
- $f = 3 \checkmark \frac{1}{2}$

- $\mathbf{n} = (-0.624242, 0.780303, 0.0380749)$ (given)
- $\mathbf{l} = \frac{(1,-8,12)-(4,-11,-6)}{\|(1,-8,12)-(4,-11,-6)\|} = \frac{1}{18.4932}(-3, 3, 18)$
- $\mathbf{l} = (-0.162221, 0.162221, 0.973329) \checkmark\checkmark$ (only $(\checkmark)$ if wrong answer)
- $\mathbf{v} = \frac{(-6,2,16)-(4,-11,-6)}{\|(-6,2,16)-(4,-11,-6)\|} = \frac{1}{27.4408}(-10, 13, 22)$
- $\mathbf{v} = (-0.36442, 0.473746, 0.801725) \checkmark\checkmark$ (only $(\checkmark)$ if wrong answer)
- $\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$
- $\mathbf{r} = (-0.16851, 0.251193, -0.953156) \checkmark\checkmark$ (only $(\checkmark)$ if wrong answer)
- $\mathbf{v} \cdot \mathbf{r} = -0.583758$, so the viewer is opposite to specular reflection. $\checkmark$

The lighting equation is then

$$\begin{aligned} I &= 0 + 0.200304 + 0.401 \times 0.745 \times (\mathbf{n} \cdot \mathbf{l}) + 0 \\ &= 0.200304 + 0.298745 \times 0.264907 \\ &= 0.200304 + 0.0791395 \\ &= 0.279444 \checkmark\checkmark\checkmark \end{aligned}$$

Total: 15

QUESTION 3: Proof

(a) **Provide** the *formula* for a ***Bézier curve of degree three (3)*** **[05]**

Solution:

A Bézier curve of degree three is given by

$$\mathbf{q}(u) = \sum_{k=0}^{3} \checkmark B_k(u) \checkmark \mathbf{p}_k,$$

where $\mathbf{p}_k$ are the control points and $B_k(u)$ are the Bernstein polynomials

$$B_k(u) = \binom{3}{k} \checkmark u^k \checkmark (1-u)^{3-k} \checkmark .$$

(b) **Show** that a ***Bézier curve of degree three (3)*** is affine invariant. **[10]**

Solution:

Let A be an affine transform, ie. $A\mathbf{x} = L\mathbf{x} + \mathbf{t}$ (where L is a linear transform). ✓✓

We must show that $A\sum_{k=0}^{3} B_k(u)\mathbf{p}_k = \sum_{k=0}^{3} B_k(u)A\mathbf{p}_k$.

$$\begin{aligned}
A\sum_{k=0}^{3} B_k(u)\mathbf{p}_k &= L\sum_{k=0}^{3} B_k(u)\mathbf{p}_k + \mathbf{t}\checkmark \\
&= \sum_{k=0}^{3} B_k(u)L\mathbf{p}_k + \mathbf{t}\checkmark \text{(Property of linear transformations)}\checkmark \\
&= \sum_{k=0}^{3} B_k(u)L\mathbf{p}_k + \sum_{k=0}^{3} B_k(u)\mathbf{t}\checkmark\checkmark \quad \text{(Property of the basis functions)}\checkmark \\
&= \sum_{k=0}^{3}(B_k(u)(L\mathbf{p}_k + \mathbf{t}))\checkmark \\
&= \sum_{k=0}^{3}(B_k(u)A\mathbf{p}_k)\checkmark
\end{aligned}$$

Total: 15

QUESTION 4: Ray Tracing Intersection

(a) Describe how to calculate the intersection point between a ray described by $\mathbf{o} + t\mathbf{d}$ (where $\mathbf{d}$ is a unit vector) and a sphere described by $(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = r^2$. **[08]**

Solution:

Let $\mathbf{p} = (x, y, z)$ and $\mathbf{c} = (c_x, c_y, c_z)$. If we use the implicit formula

$$f(\mathbf{p}) = ||\mathbf{p} - \mathbf{c}|| - r = 0 \checkmark$$

for the sphere and express any point $\mathbf{p}$ on the ray as $\mathbf{o} + t\mathbf{d}$ then we can use

$$f(\mathbf{o} + t\mathbf{d}) = ||\mathbf{o} + t\mathbf{d} - \mathbf{c}|| - r = 0 \checkmark$$

We simplify

$$\begin{aligned} f(\mathbf{o} + t\mathbf{d}) &= ||\mathbf{o} + t\mathbf{d} - \mathbf{c}|| - r = 0 \\ ||\mathbf{o} + t\mathbf{d} - \mathbf{c}|| &= r \\ (\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) &= r^2 \\ t^2(\mathbf{d} \cdot \mathbf{d}) + 2t(\mathbf{d} \cdot (\mathbf{o} - \mathbf{c})) + (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - r^2 &= 0 \checkmark\checkmark \end{aligned}$$

If $\mathbf{d}$ is normalized we can simplify even further. We can use the standard formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \checkmark\checkmark$$

To calculate t and thus the point of intersection (using $\mathbf{o}+t\mathbf{d}$), where $a = \mathbf{d}\cdot\mathbf{d}$, $b = 2\mathbf{d}\cdot(\mathbf{o}-\mathbf{c})$ and $c = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - r^2$. If $b^2 - 4ac < 0$ then there is no intersection. Since there are usually two values of t that satisfy the equation, take the smallest, positive value of t as the intersection point.✓✓

(b) How will you determine if the ray is entering or exiting the sphere? **[02]**

Solution:

If the dot product between the ray direction and surface normal is greater than zero, then the ray is exiting the surface.✓ Otherwise the ray is entering the sphere.✓

Total: 10

QUESTION 5: Rasterisation vs Ray Tracing

Discuss how ***forward shading rasterisation*** and ***naive recursive ray tracing*** *differ* in terms of application of realistic lighting. Your discussion should include how *visibility* is calculated, how reflection/refraction is computed and the relative speed of the technique.

Solution:

Students answer should be in the form of a discussion of the following table.

Criteria	**Rasterisation**	**Ray Tracing**
Visibility	Z-Buffer or Painters Algorithm✓✓	Various Intersection Tests✓✓
Lighting models	Direct lighting✓	Direct lighting✓
Shadows	Need shadow maps✓✓	Shadow rays can create basic shadows✓✓
Reflection	Need reflection renders✓✓	Reflection rays can be generated for reflections✓✓
Refraction	Need refraction renders✓✓	Refraction rays can be generated for refractions✓✓
Speed	Faster with GPU acceleration✓✓	Naive - Depends on number of objects✓✓

Total: 20

QUESTION 6: Application

A still image of an *theater stage* is to be rendered. The stage is lit by an array of spot lights, directly above the stage. Rasterisation was selected as a method for rendering the scene. Answer the questions that follow:

(a) **Provide** a *definition* for ***global lighting model***. **[05]**

> **Solution:**
> Students answer should include the fact that global lighting takes into account all paths of light, from direct✓ to indirect✓. Direct lighting comes from the light source without interacting with the scene✓. Indirect lighting is the result of light interacting with the scene in some way, usually diffuse/specular reflection or refraction (can be others but was not taught in the course)✓✓.

(b) **Discuss** how ***radiosity*** works. Your discussion must include details of patches, form factors and methods for solving radiosity as well as how radiosity can be integrated into the rasterisation process. **[15]**

> **Solution:**
> Discussion should include details about patches✓✓ and form factors✓✓ (including methods for calculating them, hemicube method✓✓✓), solving radiosity✓✓✓ using either the Jacobi method, Gauss-Seidel Iteration or shooting method. Rendering using Gouraud shading✓✓ or radiosity per pixel✓✓.

(c) **Discuss** why ***radiosity*** should be used to render the scene as opposed to ***photon mapping***. **[05]**

> **Solution:**
> Discussion should include details about the scene being mostly lit by diffuse reflections✓✓. Radiosity can only handle diffuse reflection efficiently✓. Photon mapping can handle more complex scenes but is not required for the current application✓✓.

Total: 25

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