



UNIVERSITY
OF
JOHANNESBURG

PROGRAM : BACCALAUREUS ENGINEERING
TECHNOLOGIAE
METALLURGY & CHEMICAL ENGINEERING

SUBJECT : **PROCESS CONTROL 3B**

CODE : **PRCCHB3**

DATE : SUMMER EXAMINATION 2019
13 NOVEMBER 2019

DURATION : (Y-PAPER) 08:30 - 11:30

WEIGHT : 40 : 60

TOTAL MARKS : 100

EXAMINER : MR MK KALENGA 5142

MODERATOR : LM OMARI

NUMBER OF PAGES : 2 PAGES AND 2 ANNEXURES

INSTRUCTIONS : QUESTION PAPERS MUST BE HANDED IN.

REQUIREMENTS : CALCULATORS ARE NOT REQUIRED

INSTRUCTIONS TO CANDIDATES: *Question paper to be handed in*

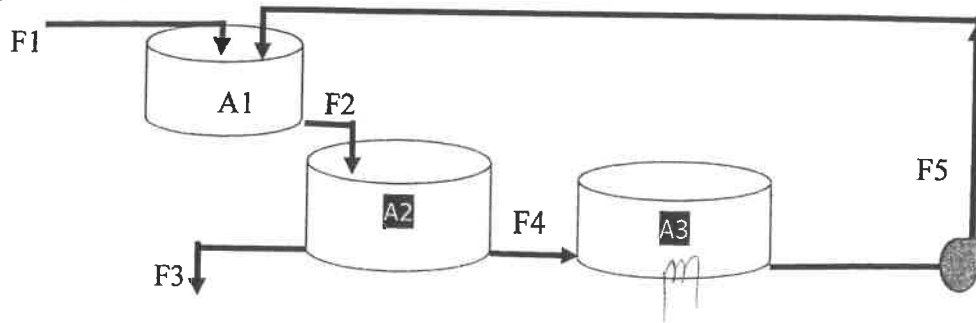
PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1

You have been provided with your BEng Tech project. Discuss the disturbances that you have identified and explain how you managed to cancel them. Be more specific. This question will be marked as per your methodology submitted through the assignment

[16]**QUESTION 2**

You are dealing with a leaching process with three tanks in series as per the sketch below. The three heights and cross-sectional areas of the three tanks are different as indicated on the sketch. It is said that all the flow rates are volumetric. The flow rate F_5 is constant and does not depend on h_3 while all other effluent flow rates are proportional to the corresponding hydrostatic liquid pressures that cause the flow.



- 1.1 Develop a mathematical model of the leaching process as above (GA1) (10)
 - 1.2 Design a COMPLETE control configuration that allows you to properly control the entire process (GA3) (10)
 - 1.3 Design the Bloc diagram of the process (10)
- [30]**

QUESTION 2 (GA1) 3

The following Laplace Transforms have been calculated and have to be assessed whether their time functions would representing a stable process or not. Use any of the methods learnt in class to discuss the stability of the process:

- 2.1 $L[e^{-at}\sin(wt)] = w/[(s+a)^2 + w^2]$
- 2.2 $L\{e^{-at}\sin(wt)\} = (s+a)/[(s+a)^2 + w^2]$

[12]

QUESTION 3

You are dealing with a process where exothermic reactions take place (producing water using Oxygen and hydrogen) in a reactor. During the process, you realize that the temperature is increasing due to the heat of reaction as it is exothermic. Due to the increase of temperature, you realize that you collect water gas and not liquid. Your aim is to produce water liquid, not gas. It is alleged that the temperature need to be decrease to satisfy your need. Making use of number of moles of reactants:

- 3.1 Establish a rough mathematical model knowing that heat needs to be removed to keep the temperature at the correct level to collect liquid water (GA1) (10)
- 3.2 Design a complete control configuration of the process (GA3) (10)
- 3.3 Design a Block diagram of the process (10)
- [30]

QUESTION 4

You are given a function whose Laplace Transform is:

$$X(s) = 1/(s-1)^3(s+3)$$

- 4.1 Calculate the time function of the above
- 4.2 Using the pole criteria to determine whether the process is stable or not, conduct the qualitative analysis of the process and comment.

[12]

TOTAL MARKS 100

$$\frac{dp}{dt} = P_1 - P_2$$

2. Compute the values of the constants C_1, C_2, \dots, C_n from eq. (8.6).
3. Find the inverse Laplace transform of every partial fraction. Then the unknown function $x(t)$ is given by

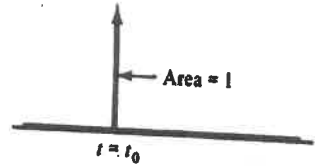
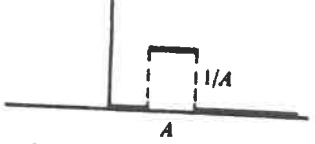
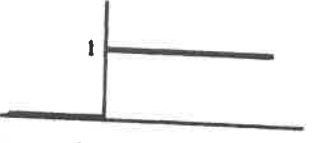
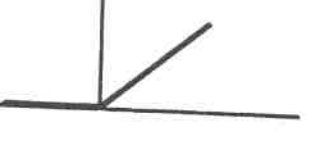
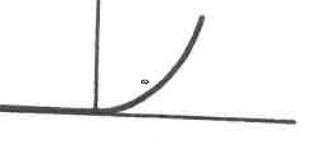


$$x(t) = \mathcal{L}^{-1} \left[\frac{C_1}{r_1(s)} \right] + \mathcal{L}^{-1} \left[\frac{C_2}{r_2(s)} \right] + \dots + \mathcal{L}^{-1} \left[\frac{C_n}{r_n(s)} \right]$$

where \mathcal{L}^{-1} symbolizes the inverse Laplace transform of the expression within the brackets. The inversion of each fraction can be done rather easily by inspection using tables of Laplace transforms for typical functions such as Tables 7.1 and 8.1.

TABLE 8.1
INVERSE LAPLACE TRANSFORMS OF SELECTED EXPRESSIONS

| Laplace transform: $f(s)$ | Time function: $f(t)$ |
|--|--|
| 1. $\frac{1}{(s+a)(s+b)}$ | $\frac{e^{-at} - e^{-bt}}{b-a}$ |
| 2. $\frac{1}{(s+a)(s+b)(s+c)}$ | $\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-b)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$ |
| 3. $\frac{s+a}{(s+b)(s+c)}$ | $\frac{1}{c-b} [(a-b)e^{-bt} - (a-c)e^{-ct}]$ |
| 4. $\frac{a}{(s+b)^2}$ | ate^{-bt} |
| 5. $\frac{a}{(s+b)^3}$ | $\frac{a}{2} t^2 e^{-bt}$ |
| 6. $\frac{a}{(s+b)^{n+1}}$ | $\frac{a}{n!} t^n e^{-bt}$ |
| 7. $\frac{1}{s(as+1)}$ | $1 - e^{-t/a}$ |
| 8. $\frac{1}{s(as+1)^2}$ | $1 - \frac{a+t}{a} e^{-t/a}$ |
| 9. $\frac{\omega^2}{s(s^2 + 2\zeta\omega s + \omega^2)}$ | $1 + \frac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}} \sin(\omega\sqrt{1-\zeta^2}t - \phi)$ where $\cos \phi = -\zeta$ |
| 10. $\frac{s}{(1+as)(s^2 + \omega^2)}$ | $-\frac{1}{1+a^2\omega^2} e^{-t/a} + \frac{1}{\sqrt{1+a^2\omega^2}} \cos(\omega t - \phi)$ where $\phi = \tan^{-1} a\omega$ |
| 11. $\frac{s}{(s^2 + \omega^2)^2}$ | $\frac{1}{2\omega} t \sin \omega t$ |
| 12. $\frac{1}{(s+a)[(s+b)^2 + \omega^2]}$ | $\frac{e^{-at}}{(a-b)^2 + \omega^2} + \frac{e^{-bt}}{\omega[(a-b)^2 + \omega^2]^{1/2}} \sin(\omega t - \phi)$ |

TABLE 7.1
LAPLACE TRANSFORMS OF VARIOUS FUNCTIONS

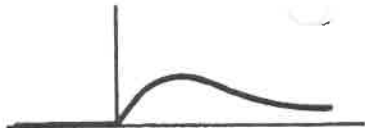

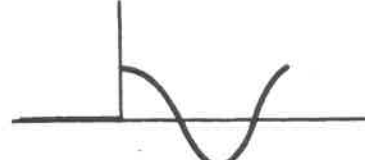
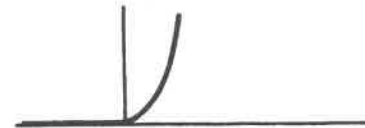
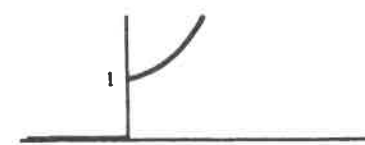


| Time function ($t \geq 0$) | Laplace transform | |
|------------------------------|-------------------------------------|---|
| Unit impulse, $\delta(t_0)$ | 1 |  |
| Unit pulse, $\delta_A(t)$ | $\frac{1}{A} \frac{1 - e^{-As}}{s}$ |  |
| Unit step | $\frac{1}{s}$ |  |
| Ramp, $f(t) = t$ | $\frac{1}{s^2}$ |  |
| t^2 | $\frac{2!}{s^3}$ |  |
| t^n | $\frac{n!}{s^{n+1}}$ |  |
| e^{-at} | $\frac{1}{s+a}$ |  |

Addition

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

TABLE 7.1 (cont.)

| | | |
|--------------------------|-------------------------------------|---|
| $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ |  |
| $\sin(\omega t)$ | $\frac{\omega}{s^2 + \omega^2}$ |  |
| $\cos(\omega t)$ | $\frac{s}{s^2 + \omega^2}$ |  |
| $\sinh(\omega t)$ | $\frac{\omega}{s^2 - \omega^2}$ |  |
| $\cosh(\omega t)$ | $\frac{s}{s^2 - \omega^2}$ |  |
| $e^{-at} \sin(\omega t)$ | $\frac{\omega}{(s+a)^2 + \omega^2}$ |  |
| $e^{-at} \cos(\omega t)$ | $\frac{(s+a)}{(s+a)^2 + \omega^2}$ |  |

7.3 Laplace Transforms of Derivatives

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = s\bar{f}(s) - f(0) \quad (7.14)$$

where $\bar{f}(s) = \mathcal{L}[f(t)]$.

Proof:

$$\begin{aligned} \mathcal{L}\left[\frac{df(t)}{dt}\right] &= \int_0^\infty \frac{df(t)}{dt} e^{-st} dt = [e^{-st} f(t)]_0^\infty + \int_0^\infty s e^{-st} f(t) dt \\ &= [0 - f(0)] + s \int_0^\infty f(t) e^{-st} dt = s\bar{f}(s) - f(0) \end{aligned}$$

Similarly, it can be proved that

$$\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 \bar{f}(s) - s f(0) - f'(0) \quad (7.15)$$

where $f'(0) = df(t)/dt$ evaluated at $t = 0$. In general,

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0) \quad (7.16)$$

From eqs. (7.14) through (7.16) we notice that in order to find the Laplace transform of any derivative, we need to have a number of initial conditions. To find the Laplace transform of an n th-order derivative, we need n initial conditions,

$$f(0), f'(0), f''(0), \dots, f^{(n-1)}(0)$$

7.4 Laplace Transforms of Integrals

$$\mathcal{L}\left[\int_0^t f(t) dt\right] = \frac{1}{s} \bar{f}(s) \quad (7.17)$$

where $\bar{f}(s) = \mathcal{L}[f(t)]$.

Proof:

$$\mathcal{L}\left[\int_0^t f(t) dt\right] = \int_0^\infty \left[\int_0^t f(t) dt\right] e^{-st} dt$$