PHYE0A2/PHYE2A2 2020 (v1.2c): Formulas and Identities

List of Definitions

- $\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$
- $\vec{\mathbf{B}} = \mu_0(\vec{\mathbf{H}} + \vec{\mathbf{M}}) = \mu\vec{\mathbf{H}}$
- $C = Q/V$
- $\vec{\mathbf{D}} = \varepsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}} = \varepsilon\vec{\mathbf{E}}$
- $\varepsilon = \varepsilon_0\varepsilon_r$
- $\varepsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{F/m}$
- $\varepsilon_r = 1 + \chi_e$
- $\vec{\mathbf{J}}_{bs} = \vec{\mathbf{M}} \times \hat{\mathbf{n}}$
- $\vec{\mathbf{J}}_{bv} = \nabla \times \vec{\mathbf{M}}$
- $k = 2\pi/\lambda$
- $\vec{\mathbf{M}} = \chi_m\vec{\mathbf{H}}$
- $\vec{\mathbf{m}} = I\vec{\mathbf{A}}$; $\vec{\mathbf{m}} = \int \vec{\mathbf{M}}\mathrm{d}\tau$
- $\mu_0 \approx 1.26 \times 10^{-6}\,\mathrm{H/m}$
- $\mu_r = 1 + \chi_m$
- $\vec{\mathbf{P}} = N\vec{\mathbf{p}}$
- $\vec{\mathbf{P}} = \varepsilon_0\chi_e\vec{\mathbf{E}}$
- $\vec{\mathbf{p}} = Q\vec{\mathbf{s}}$
- $\rho = \rho_f + \rho_b$
- $\rho_b = -\nabla \cdot \vec{\mathbf{P}}$
- $\vec{\mathbf{R}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}'$
- $\sigma_b = \vec{\mathbf{P}} \cdot \hat{\mathbf{n}}$
- $v = \lambda f$
- $\omega = 2\pi f$

List of Equations

- $\vec{\mathbf{A}} = \frac{\mu}{4\pi}\int_\tau \frac{\vec{\mathbf{J}}_v}{R}\mathrm{d}\tau'$;
  $\vec{\mathbf{A}} = \frac{\mu}{4\pi}I\int_c \frac{1}{R}\mathrm{d}\vec{\ell}'$;
  $\vec{\mathbf{A}} = \frac{\mu}{4\pi}\int_\tau \frac{\vec{\mathbf{M}}\times\hat{\mathbf{R}}}{R^2}d\tau'$
- $\vec{\mathbf{B}} = \frac{\mu}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{R}}}{R^2}$;
  $\vec{\mathbf{B}} = \frac{\mu}{4\pi}\int_c \frac{I\mathrm{d}\vec{\ell}'\times\hat{\mathbf{R}}}{R^2}$;
  $\vec{\mathbf{B}} = \frac{\mu}{4\pi}\int_\tau \frac{\vec{\mathbf{J}}_v\times\hat{\mathbf{R}}}{R^2}\mathrm{d}\tau'$
- $(\vec{\mathbf{B}}_1 - \vec{\mathbf{B}}_2)\cdot\hat{\mathbf{n}} = B_{n1} - B_{n2} = 0$
- $\nabla \cdot \vec{\mathbf{B}} = 0$
- $\oint_s \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{a}} = 0$
- $(\vec{\mathbf{D}}_1 - \vec{\mathbf{D}}_2)\cdot\hat{\mathbf{n}} = D_{n1} - D_{n2} = \sigma_f$
- $\oint_s \vec{\mathbf{D}} \cdot \mathrm{d}\vec{\mathbf{s}} = \int_\tau \rho_f\mathrm{d}\tau$
- $\nabla \cdot \vec{\mathbf{D}} = \rho_f$
- $\vec{\mathbf{E}} = -\nabla V$
- $\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon R^2}\hat{\mathbf{R}}$;
  $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon}\int_\tau \frac{\rho\hat{\mathbf{R}}}{R^2}\mathrm{d}\tau$
- $(\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2)\times\hat{\mathbf{n}} = \vec{\mathbf{E}}_{t1} - \vec{\mathbf{E}}_{t2} = 0$
- $\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon}$
- $\nabla \times \vec{\mathbf{E}} = -\frac{\partial\vec{\mathbf{B}}}{\partial t}$
- $\nabla^2\vec{\mathbf{E}} = \mu\varepsilon\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2}$
- $\oint_c \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\ell} = -\int_s \frac{\partial\vec{\mathbf{B}}}{\partial t}\cdot\mathrm{d}\vec{\mathbf{s}}$
- $\oint_s \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{s}} = \frac{1}{\varepsilon}\int_\tau \rho\mathrm{d}\tau$
- $\vec{\mathbf{F}}_c = \frac{mv^2}{r}\hat{\mathbf{r}}$
- $\vec{\mathbf{F}}_{ab} = \frac{1}{4\pi\epsilon}\frac{Q_aQ_b}{r^2}\hat{\mathbf{r}}$
- $\vec{\mathbf{F}}_{EM} = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$
- $\vec{\mathbf{F}}_M = I\int_c \mathrm{d}\vec{\ell}\times\vec{\mathbf{B}}$
- $\vec{\mathbf{F}}_{M2} = -\frac{\mu_0 I_1 I_2}{4\pi}\int_{c1}\oint_{c2}\frac{\mathrm{d}\vec{\ell}_1\cdot\mathrm{d}\vec{\ell}_2}{R_{21}^3}\vec{\mathbf{R}}_{21}$
- $(\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2)\times\hat{\mathbf{n}} = \vec{\mathbf{H}}_{t1} - \vec{\mathbf{H}}_{t2} = \vec{\mathbf{J}}_{bs}$
- $\oint_c \vec{\mathbf{H}}\cdot\mathrm{d}\ell = I_f + \int_s \frac{\partial\vec{\mathbf{D}}}{\partial t}\cdot\mathrm{d}\vec{\mathbf{s}}$
- $\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}}_f + \frac{\partial\vec{\mathbf{D}}}{\partial t}$
- $I = \int_s \vec{\mathbf{J}}\cdot\mathrm{d}\vec{\mathbf{s}}$
- $\vec{\mathbf{J}} = nq\vec{\mathbf{v}}$
- $\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$
- $\nabla\cdot\vec{\mathbf{J}} = -\frac{\partial\rho}{\partial t}$
- $(\vec{\mathbf{J}}_1 - \vec{\mathbf{J}}_2)\cdot\hat{\mathbf{n}} = J_{n1} - J_{n2} = 0$
- $(\vec{\mathbf{J}}_1/\sigma_1 - \vec{\mathbf{J}}_2/\sigma_2)\times\hat{\mathbf{n}} = \vec{\mathbf{J}}_{t1}/\sigma_1 - \vec{\mathbf{J}}_{t2}/\sigma_2 = 0$
- $L = N\frac{\mathrm{d}\Phi}{\mathrm{d}i}$
- $M_{12} = N_1\frac{\mathrm{d}\Phi_{12}}{\mathrm{d}i_2}$
- $\vec{\mathbf{P}} = \vec{\mathbf{S}} = \vec{\mathbf{E}}\times\vec{\mathbf{H}}$
- $< P > = \frac{1}{T}\int_0^T |\vec{\mathbf{P}}|\mathrm{d}t$
- $\Phi_B = \int_s \vec{\mathbf{B}}\cdot\mathrm{d}\vec{\mathbf{s}}$; $\Phi_E = \int_s \vec{\mathbf{E}}\cdot\mathrm{d}\vec{\mathbf{s}}$
- $\rho_b = -\left(\frac{\epsilon_r - 1}{\epsilon_r}\right)\rho_f$
- $R = \rho\frac{l}{A}$
- $u_E = \frac{1}{2}\epsilon E^2$; $u_M = \frac{1}{2}\mu H^2$
- $V_1 - V_2 = \int_1^2 \vec{\mathbf{E}}\cdot\mathrm{d}\vec{\ell}$
- $V = \frac{Q}{4\pi\epsilon R}$;
  $V = \frac{\vec{\mathbf{p}}\cdot\hat{\mathbf{R}}}{4\pi\epsilon R^2}$;
  $V = \frac{1}{4\pi\epsilon}\int_\tau \frac{\rho\mathrm{d}\tau}{R}$
- $V_i = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -L\frac{\mathrm{d}i}{\mathrm{d}t}$
- $W = \frac{1}{2}CV^2$

Cartesian coordinates

- $d\vec{\ell} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$
- $d\vec{\mathbf{s}}_x = dydz\hat{\mathbf{x}}$
- $d\vec{\mathbf{s}}_y = dzdx\hat{\mathbf{y}}$
- $d\vec{\mathbf{s}}_z = dxdy\hat{\mathbf{z}}$
- $d\tau = dxdydz$
- $\nabla = \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}$
- $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
- $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$; $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$; $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$

Cylindrical coordinates

- $x = \rho\cos\phi$; $y = \rho\sin\phi$; $z = z$
- $A_\rho = A_x\cos\phi + A_y\sin\phi$
  $A_\phi = -A_x\sin\phi + A_y\cos\phi$
  $A_z = A_z$
- $d\vec{\ell} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{\mathbf{z}}$
- $d\vec{\mathbf{s}}_\rho = \rho d\phi dz\hat{\rho}$
  $d\vec{\mathbf{s}}_\phi = d\rho dz\hat{\phi}$
  $d\vec{\mathbf{s}}_z = \rho d\rho d\phi\hat{\mathbf{z}}$
- $d\tau = \rho d\rho d\phi dz$
- $\nabla\cdot\vec{\mathbf{F}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho F_\rho] + \frac{1}{\rho}\frac{\partial}{\partial\phi}[F_\phi] + \frac{\partial}{\partial z}[F_z]$
- $\hat{\rho}\times\hat{\phi} = \hat{\mathbf{z}}$; $\hat{\phi}\times\hat{\mathbf{z}} = \hat{\rho}$; $\hat{\mathbf{z}}\times\hat{\rho} = \hat{\phi}$

Spherical coordinates

- $x = r\cos\phi\sin\theta$; $y = r\sin\phi\sin\theta$; $z = r\cos\theta$
- $A_r = A_x\cos\phi\sin\theta + A_y\sin\phi\sin\theta + A_z\cos\theta$
  $A_\theta = A_x\cos\phi\cos\theta + A_y\sin\phi\cos\theta - A_z\sin\theta$
  $A_\phi = -A_x\sin\phi + A_y\cos\phi$
- $d\vec{\ell} = dr\hat{\mathbf{r}} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$
- $d\vec{\mathbf{s}}_r = r^2\sin\theta d\theta d\phi\hat{\mathbf{r}}$
  $d\vec{\mathbf{s}}_\theta = rdr\sin\theta d\phi\hat{\theta}$
  $d\vec{\mathbf{s}}_\phi = rdrd\theta\hat{\phi}$
- $d\tau = r^2 dr\sin\theta d\theta d\phi$
- $\nabla\cdot\vec{\mathbf{F}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2F_r\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta F_\theta] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[F_\phi]$
- $\hat{\mathbf{r}}\times\hat{\theta} = \hat{\phi}$; $\hat{\theta}\times\hat{\phi} = \hat{\mathbf{r}}$; $\hat{\phi}\times\hat{\mathbf{r}} = \hat{\theta}$

Integrals

- $\int\frac{1}{(a^2+x^2)^{1/2}}dx = \ln\left(\sqrt{a^2+x^2}+x\right) + C$
- $\int\frac{x}{(a^2+x^2)^{1/2}}dx = \sqrt{a^2+x^2} + C$
- $\int\frac{1}{(a^2+x^2)^{3/2}}dx = \frac{x}{a^2(a^2+x^2)^{1/2}} + C$
- $\int\frac{x}{(a^2+x^2)^{3/2}}dx = -\frac{1}{(a^2+x^2)^{1/2}} + C$
- $\int\frac{x^2}{(a^2+x^2)^{3/2}}dx = \ln\left(\sqrt{a^2+x^2}+x\right) - \frac{x}{(a^2+x^2)^{1/2}} + C$

Identities

- $(\vec{\mathbf{A}}\times\vec{\mathbf{B}})\cdot\vec{\mathbf{C}} = \vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}\times\vec{\mathbf{C}})$
- $\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}}) = \vec{\mathbf{B}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}) - \vec{\mathbf{C}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})$
- $\nabla(fg) = f\nabla g + g\nabla f$
- $\nabla\cdot(f\vec{\mathbf{A}}) = (\nabla f)\cdot\vec{\mathbf{A}} + f(\nabla\cdot\vec{\mathbf{A}})$
- $\nabla\cdot(\vec{\mathbf{A}}\times\vec{\mathbf{B}}) = \vec{\mathbf{B}}\cdot(\nabla\times\vec{\mathbf{A}}) - \vec{\mathbf{A}}\cdot(\nabla\times\vec{\mathbf{B}})$
- $\nabla\times(f\vec{\mathbf{A}}) = (\nabla f)\times\vec{\mathbf{A}} + f(\nabla\times\vec{\mathbf{A}})$
- $\nabla(\nabla\cdot\vec{\mathbf{A}}) = \nabla^2\vec{\mathbf{A}} + \nabla\times\nabla\times\vec{\mathbf{A}}$
- $\nabla(1/R) = -\hat{\mathbf{R}}/R^2$
- $\nabla'(1/R) = \hat{\mathbf{R}}/R^2$
- $\oint_s\vec{\mathbf{A}}\cdot d\vec{\mathbf{s}} = \int_\tau\boldsymbol{\nabla}\cdot\vec{\mathbf{A}}d\tau$
- $\int_s(\boldsymbol{\nabla}\times\vec{\mathbf{A}})\cdot d\vec{\mathbf{s}} = \oint_c\vec{\mathbf{A}}\cdot d\vec{\ell}$