Part 1 Question 1- 10 marks (1 version)

Question	The electric field intensity in a sphere with radius b is given by $\vec{E}(r) = \frac{42}{\varepsilon_0}(4\ b\ r^2 - b^3 e^{2r/b})\hat{r}$ $C.m^{-3}$. a. (2 pts) Determine the charge distribution within the sphere. b. (2 pts) Determine the electric field intensity outside the sphere. c. (6 pts) Derive the electric potenial functions for inside and outside the sphere.
Instructor Notes	This question took me 17 minutes to complete (4+5+8)

Part 1 Question 2 – 10 marks (2 versions)

Question	A metal sphere of radius a has a uniform (free) charge density σ_f on its surface. The permittivity of the dielectric region surrounding the sphere varies as $\varepsilon(r) = 30\varepsilon_0 (\frac{r+a}{r})$, where r is the radial coordinate. a. (1 pts) Determine the polarization P and electric field intensity E inside the sphere. b. (3 pts) Determine the polarization P and electric field intensity E in the dielectric. c. (5 pts) Calculate all bound charge densities, ρ_b and σ_b. Is the dielectric homogeneous? d. (1 pts) Test whether the electric field intensity E in the dielectric is conservative.
Categories	Electrostatics
Topics	dielctric surface charge
Instructor Notes	Took me 15 minutes to complete this question (1+5+8+1).

Question	A metal sphere of radius a has a uniform (free) charge density σ_f on its surface. The permittivity of the dielectric region surrounding the sphere varies as $\varepsilon(r) = 30\varepsilon_0 (\frac{2r-a}{r})$, where r is the radial coordinate. a. (1 pts) Determine the polarization P and electric field intensity E inside the sphere. b. (3 pts) Determine the polarization P and electric field intensity E in the dielectric. c. (5 pts) Calculate all bound charge densities, ρ_b and σ_b. Is the dielectric homogeneous? d. (1 pts) Test whether the electric field intensity E in the dielectric is conservative.
Categories	Electrostatics
Topics	dielctric surface charge
Instructor Notes	Took me 15 minutes to complete this question (1+5+8+1).

Part 1 Question 3 – 6 marks (2 versions)

Question	The magnitude of the current density in a medium (σ = 20 $S.m^{-1}$, $\varepsilon_r = 4.5$) is J = 15 $A.m^{-2}$. The current density vector makes an angle of 45° with respect to the normal at an interface to a different medium (σ = 10 $S.m^{-1}$, $\varepsilon_r = 4.0$). a. (1 pts) Calculate the normal and tangential components of the current density in the first medium. b. (2 pts) Calculate the normal and tangential components of the current density in the other medium. c. (2 pts) Calculate the angle to the normal of the current density in the second medium. d. (3 pts) Calculate the surface charge density at the interface.
Categories	Current Electricity
Topics	Boundary conditions
Instructor Notes	It took me 12 minutes to finish this question (3+2+3+4).

Question	A system consists of two conducting materials. In material 1 (σ = 300 $S.m^{-1}$, $\varepsilon_r = 13.5$) we measure a current density of J = 18 $A.m^{-2}$. The current density vector makes an angle of 33° with respect to the normal at an interface to the second medium (σ = 10 $S.m^{-1}$, $\varepsilon_r = 4.0$). a. (1 pts) Calculate the normal and tangential components of the current density in the first medium. b. (2 pts) Calculate the normal and tangential components of the current density in the other medium. c. (2 pts) Calculate the angle to the normal of the current density in the second medium. d. (3 pts) Calculate the surface charge density at the interface.
Categories	Current Electricity
Topics	Boundary conditions
Instructor Notes	It took me 12 minutes to finish this question (3+2+3+4).

Part 2 Question 1 – 6 marks (2 versions)

Question

A very long straight conductor located along the z-axis has a circular cross section with a radius of 10 cm. The conductor carries a current of 100 A in the positive z-direction. The non-uniform current density is described by the equation $\vec{J}(\rho) = k\,\rho\,\hat{z}$, where k is a constant and ρ is the radial coordinate.

a. (2 pts) Using the given current density, find the magnetic field intensity H inside the conductor.
b. (1 pts) Calculate the magnetic field intensity H outside the conductor.
c. (2 pts) Determine for the above mentioned current the constant *k*.
d. (1 pts) Sketch the field intensity H as a function of the radial coordinate ρ. What (and where) is the maximum of H?

Question

A very long straight conductor located along the z-axis has a circular cross section with a diameter of 10 cm. The conductor carries a current of 100 A in the positive z-direction. The non-uniform current density is described by the equation $\vec{J}(\rho) = k\,\rho^2\,\hat{z}$, where k is a constant and ρ is the radial coordinate.

a. (2 pts) Using the given current density, find the magnetic field intensity H inside the conductor.
b. (1 pts) Calculate the magnetic field intensity H outside the conductor.
c. (2 pts) Determine for the above mentioned current the constant *k*.
d. (1 pts) Sketch the field intensity H as a function of the radial coordinate ρ. What (and where) is the maximum of H?

Part 2 Question 2 – 10 marks (4 versions)

Question

Two very long straight wires are aligned parallel with a separation of *d*. Both wires have a finite radius *r*. The wires carry currents I_1 and I_2, respectively. The wires lie in the *xz*-plane parallel to the *z*-axis.

a. (3 pts) Calculate the magnetic flux density produced by the current *I* in one straight long wire using the Biot-Savart law. Explain your steps.
b. (4 pts) Calculate the force per unit length between the wires. Show under which conditions the force is repulsive.
c. (4 pts) Determine the total magnetic field intensity in the plane spanned by the wires (*xz*-plane). Determine all roots, minima and maxima of the field in this plane if $I_2 = 2\,I_1$.

Question

Two very long straight wires are aligned parallel with a separation of *d*. Both wires have a finite radius *r*. The wires carry currents I_1 and I_2, respectively. The wires lie in the *xz*-plane parallel to the *z*-axis.

a. (3 pts) Calculate the magnetic flux density produced by the current *I* in one straight long wire using the Biot-Savart law. Explain your steps.
b. (4 pts) Calculate the force per unit length between the wires. Show under which conditions the force is attractive.
c. (4 pts) Determine the total magnetic field intensity in the plane spanned by the wires (*xz*-plane). Determine all roots, minima and maxima of the field in this plane if $I_2 = -0.5\,I_1$.

Question

Two regions of magnetic materials are separated by the z=0 plane. In the region with z>0, the magnetic field intensity is given by $\vec{H} = 40\,\hat{x} + 50x\hat{y} + 12\hat{z}$ kA per m. The relative permeability in this region is 20.

a. (5 pts) Determine the magnetic field intensity in the region with z<0, which has a relative permeability of 100. Take into account a surface current density of $\vec{J} = 10\hat{y}$ kA per m at z=0.
b. (4 pts) Calculate the bound surface charge density and determine whether the materials are homogeneous.

Question

In the region situated at z>0, the magnetic field intensity is given by $\vec{H} = 40y\,\hat{x} + 50\hat{y} + 12\hat{z}$ A per m. The relative permeability in this region is 100.

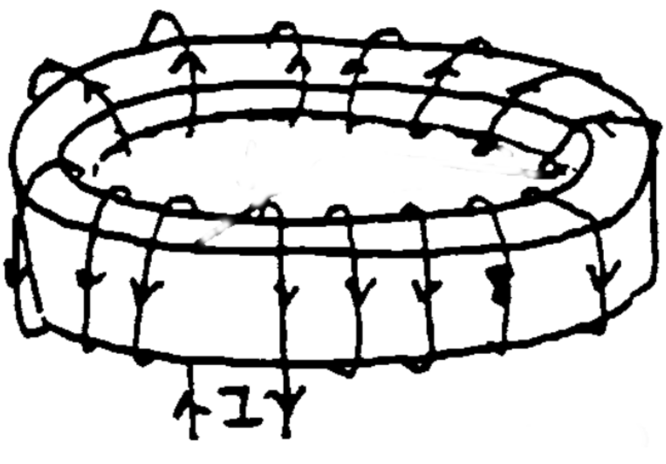
a. (5 pts) Determine the magnetic field intensity in the region with z<0, which has a relative permeability of 20. Take into account a surface current density of $\vec{J} = 30\hat{x}$ A per m at z=0.
b. (4 pts) Calculate the bound surface charge density and determine whether the materials are homogeneous.

Part 2 Question 3 – 6 marks (2 versions)

Question

A toroid of rectangular cross section is wound with N turns, as shown in the figure. The toroid has an inner radius *a*, an outer radius *b* and a height *h*. The toroid has a core made of a material with $\mu_r \neq 1$.

a. (2pts) Show that the magnetic flux density $\vec{B}$ generated by the coil is given by $\vec{B}(\rho) = \frac{\mu}{2\pi} IN \frac{1}{\rho}\hat{\phi}$ for a ≤ ρ ≤ b.
b. (3 pts) Compute the self-inductance of the toroid.
c. (1 pt) If the current in the coil varies as I(t) = I_0 cos(wt), determine the induced voltage in the coil.



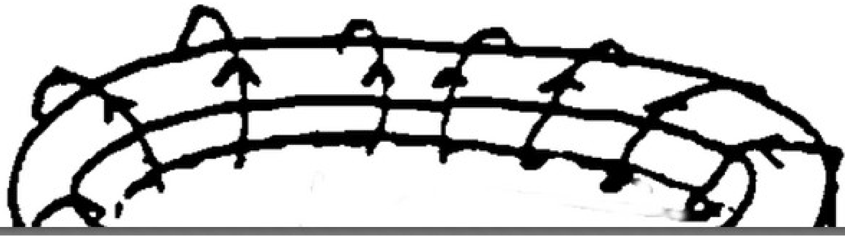
Question

A toroid of square cross section is wound with N turns, as shown in the figure. The toroid has a core with a magnetic material.

a. (2 pts) Show that the magnetic field intensity $\vec{H}$ generated by the coil is given by $\vec{H}(\rho) = \frac{1}{2\pi} IN \frac{1}{\rho} \hat{\phi}$ inside the coil

b. (3 pts) Compute the self-inductance of the toroid.

c. (1 pts) If the current in the coil varies as $I(t) = \omega \sin(\omega t)$, determine the induced voltage in the coil.



Part 2 Question 4 – 10 marks (2 versions)

Question

The magnetic field in a dielectric medium is given by

$\vec{H} = H_0 \sin(\omega t - kx)\, \hat{z}$

.

a. (4pts) Compute the B, D and E field.

b. (3pts) Derive the relation between ω and k given by the Maxwell equations.

c. (3pts) Compute the electric and magnetic energy density.

Question

The electric field in a dielectric medium is given by

$\vec{E} = -E_0 \cos(-\omega t)\sin(bx)\, \hat{y}$

.

a. (3pts) Compute the magnetic flux density $\vec{B}$.

b. (4pts) Derive the relation between ω and b given by the Maxwell equations.

c. (3pts) Compute the Poynting vector and the average power density.