University of Johannesburg
PHY8X17: Mathematical Methods
Big maybe-final assessment exam thingy (02 June 2020)

Examiner: Dr. R. Warmbier
Moderator: Prof. A. Chen (Wits)
Time: 150 minutes + 60 minutes for submission
Available Marks: 55

The usual exam rules of the University of Johannesburg apply. This is an individual assessment. You may use the equation sheet and the lecture notes. No other sources are allowed without explicit permission by the lecturer.

---

1. Find the complete series solution for the ordinary differential equation $y'' + \frac{1}{x^2}y' - \frac{2}{x^2}y = 0$. [7]

2. Solve the ordinary differential equation $\frac{\mathrm{d}^2\phi}{\mathrm{d}\omega^2} + m\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = e^{-\omega}$ using the Green's function method for $\omega \geq 0$. Observe the initial conditions $\phi(0) = \phi'(0) = 0$. [12]

3. Determine the integral $\int_0^\infty \frac{\mathrm{d}x}{1+x^2}$ using calculus of residues. [6]

4. Determine the Fourier series for the function $f(x) = x^3$ for $-\pi < x < \pi$.
   Would the result differ, if the intervall was $0 < x < 2\pi$? Explain. [7]

5. A number of physics problems involve terms of the form $\frac{1}{(\omega_0 - \omega) - i\gamma}$. Compute its inverse Fourier transform for $\gamma > 0$. [7]

6. Derive the Green's function $G(\vec{r}, \vec{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}$ for the Poisson equation $\nabla^2\Phi = \frac{\rho}{\epsilon_0}$ using Fourier transforms. Do not use knowledge about Coulomb potential.
   You may use $\mathrm{FT}[\frac{1}{k^2}] = \left(\frac{\pi}{2}\right)^{1/2}\frac{1}{r}$. [5]

7. The Lagrangian of an electromagnetic field with charge density ρ and current density $\vec{\mathbf{J}}$ is given by $L = \frac{1}{2}\left(\varepsilon\vec{\mathbf{E}}^2 - \frac{1}{\mu}\vec{\mathbf{B}}^2\right) - \rho\phi + \vec{\mathbf{A}} \cdot \vec{\mathbf{J}}$ with $\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$ and $\vec{\mathbf{E}} = -\nabla\phi - \frac{\partial\vec{\mathbf{A}}}{\partial t}$. Use these three equations to derive all four Maxwell equations. [11]

—END OF PAPER—