University of Johannesburg
Department of Physics
PHY8X03: Particle physics 1
Exam 1 – Version A (XX June 2020)
Time: 120 minutes

UNIVERSITY OF JOHANNESBURG

Student No.: ____________________ **Surname, Initials:** ____________________

Instructions:

- Enter your Surname, Initials and Student # above, and include this coversheet as a coversheet to your test when you submit it.
- Read all questions and instructions carefully. It is your responsibility to make sure that your paper has 13 pages (excluding the coversheet(s)).
- Answer the written questions on the question paper in pen ONLY.

Written Marks
Tot. ________/100
Mark Summary
5 Written: 100^{pts}
Full-marks: 100^{pts}

Written Questions

- **Draw** diagrams where appropriate.
  Marks are allocated for diagrams.
- **Show** all work, clearly and in order, if you want to get full credit. **Justify** the steps you take to ensure full marks. We reserve the right to take off marks if we cannot see how you arrived at your answer (even if your final answer is correct). Please keep your written answers **brief**; be clear and to the point. We reserve the right to take points off for rambling, incorrect or irrelevant statements.
- Do algebra with variables. Numerical values can be substituted at the end. Numerical work will only be evaluated at the last step.
- Underline or otherwise indicate your final answers.

Fundamental constants and useful numbers

Quantity	**Symbol**	**Value**
speed of light in vacuum	c_0	$3.00 \times 10^{8}\,\mathrm{m/s}$
Magnitude of electron charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$1.6726 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$1.6749 \times 10^{-27}\,\mathrm{kg}$

Written Questions 100 *points* (100.0% of available marks)

1. [10 pts]

(a) (2 pts) Find the mass density of a proton, modelling it as a solid sphere of radius $1.00 \times 10^{-15} m$.

(b) (2 pts) Consider a classical model of an electron as a solid sphere with the same density as the proton. Find its radius.

(c) (4 pts) Imagine that this electron possesses spin angular momentum $I\omega = \hbar/2$, because of classical rotation about the z-axis. Determine the speed of a point on the equator of the electron

(d) (2 pts) Now compare this speed to the speed of light and discuss.

2.

10 pts

Determine if a conservation law is violated for each reaction

(a) (2 pts) $B \rightarrow \mu^+ + \mu^-$, where B is the B-meson

(b) (2 pts) $p + \tau^- \rightarrow \tau^+ + p$

(c) (2 pts) $p + p \rightarrow \pi^+ + p$

(d) (2 pts) $p + n \rightarrow p + p + n$

(e) (2 pts) $p + \gamma \rightarrow \pi^0 + n$

3.

20 pts

The Lagrangian for an interacting complex scalar field ϕ is:

$$\mathcal{L} = (\mathcal{D}_\mu \phi)^* (\mathcal{D}^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

(a) (8 pts) Show that this is invariant under the local gauge transformation $\phi \to e^{i\theta(x)/v}\phi$ provided that $A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu \theta$.

(b) (12 pts) Writing $\phi = \frac{1}{\sqrt{2}}\,(v + h(x))\,e^{i\theta(x)/v}$ with h real and $v = \sqrt{-\mu^2/\lambda}$, show that the Lagrangian in question 3.(a) can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \mu^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu A^\mu + \frac{\mu^4}{4\lambda} - \lambda v h^3 - \frac{\lambda}{4}h^4$$
$$+\frac{1}{2}e^2 A_\mu A^\mu h^2 + e^2 v A_\mu A^\mu h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

4.

40 pts

Consider a scalar QED with Higgs phenomena system using the Lagrangian

$$\mathcal{L} = (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) + \frac{\mu^2}{2} \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with $\mathcal{D}_\mu \phi = (\partial_\mu - ieA_\mu)\,\phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Consider the static case where $\partial^0 \phi = \partial^0 \mathbf{A} = 0$ and $A_0 = 0$. Recall that $\epsilon^{ijk} B^k = -F^{ij}$ and $E^i = -F^{0i}$ in units where $c = 1$.

(a) (13 pts) Show that the equation of motion for $\mathbf{A}$ is of the form

$$\nabla \times \mathbf{B} = \mathbf{J} \;\text{ with }\; \mathbf{J} = ie \left[\phi^\dagger (\nabla - ie\mathbf{A})\phi - (\nabla + ie\mathbf{A})\phi^\dagger \phi \right] .$$

(b) (20 pts) Show that with spontaneous symmetry breaking, in the classical approximation $\phi = v = \sqrt{\mu^2/\lambda}$, the current $\mathbf{J}$ is of the form

$$\mathbf{J} = e^2 v^2 \mathbf{A} \quad \text{(the London equation)}$$

and thus $\nabla^2 \mathbf{B} = e^2 v^2 \mathbf{B}$, the Meissner effect (what solution to $\mathbf{B}$ does this imply, and discuss). Recall that $\nabla \times (\nabla \phi) = 0$, $\nabla \cdot (\nabla \times \mathbf{D}) = 0$, $\nabla^2 \psi = \nabla \cdot (\nabla \psi)$ and $\nabla \times (\nabla \times \mathbf{D}) = \nabla(\nabla \cdot \mathbf{D}) - \nabla^2 \mathbf{D}$.

(c) (7 pts) The resistivity ρ for the system is defined by

$$\mathbf{E} = \rho \mathbf{J} \ .$$

Show that, in this case of spontaneous symmetry breaking, $\rho = 0$, and we have superconductivity.

5.

20 pts

Consider the case of one Hermitian scalar field ϕ with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi^\dagger - V_a(\phi)\ ,$$

with scalar potential

$$V_a(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4\ .$$

You have seen that this potential has a degenerate minimum at $\phi = \pm v$, with $v = \sqrt{\mu^2/\lambda}$. Suppose we add a cubic term to $V_a(\phi)$

$$V_b(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{2\xi}{3}\phi^3 + \frac{\lambda}{4}\phi^4\ .$$

Show that the degeneracy in the minimum of $V_a(\phi)$ is now removed. Find the true minimum of $V_b(\phi)$. Also, show that, as a function of the parameter ξ, the VEV $\langle\phi\rangle_0$ changes discontinuously from $\langle\phi\rangle_0 = -v$ to $\langle\phi\rangle_0 = v$ as ξ changes from positive to negative values going through 0.