UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

	Examiner	Moderator
Marks		

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING:
ELECTRICAL/MECHANICAL/INDUSTRIAL ENGINEERING

CAMPUS: **DFC**

MODULE: **MAT3AW3** ENGINEERING MATHEMATICS 3

22 MAY 2020

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MARKS	**100**

NUMBER OF PAGES: 3

REQUIREMENTS **:** INFORMATION BOOKLET
NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS : Please write your particulars on all pages of your answer sheet.
Answer all the questions.

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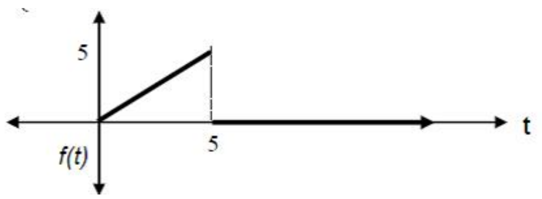
1. (a) Sketch the graph of the function $f(t) = (1 - t^2)[H(t-1) - H(t-3)]$ for $t \geq 0$. [3]

(b) Evaluate $\frac{1}{D^2+7}\{\cos\sqrt{7}t + e^{-\pi t}\}$ [4]

(c) The function represented by the graph below is defined analytically as

$$f(t) = \begin{cases} t & 0 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$



(i) Express $f(t)$ in terms of Heaviside functions. [2]
(ii) Find $L\{f(t)\}$ [4]

2. Determine the unique solutions of the following differential equations by using the **Laplace transform**, subject to the indicated initial conditions:

(a) $y'' - 4y = 4t, \quad y(0) = 0, y'(0) = -1$ [7]

(b) $y'' - 4y' - 5y = 2\delta(t-2), \quad y(0) = -1, y'(0) = 0$ [8]

(c) $y'' + 2y' - 3y = e^{-3(t-2)}H(t-2) \quad y(0) = 1, y'(0) = 1,$ [11]

3. Find the general solutions of the following differential equations, using **D-operator methods**.

(a) $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} = 5\cos 3t - \frac{2}{e^t},$ [8]

(b) $(D^2 + D - 2)y = x^2e^{3x},$ [7]

4. The equation governing the build up of charge, $q(t)$, on the capacitor of an RC circuit is

$$R\frac{dq}{dt} + \frac{1}{C}q = v_0$$

where v_0 is the constant d.c. voltage. Initially, the circuit is relaxed and the circuit 'closed' at $t = 0$ and so $q(0) = 0$ is the initial condition for the charge. Let $R = 0.5, c = 2$ and $v_0 = 10$.

(a) Use the **Laplace transform** to solve the differential equation for $q(t)$. [7]

(b) Use **D-operator methods** to solve the differential equation for $q(t)$. [5]

5. Use **D-operator methods** to solve the following system of differential equations for x and y,

$$\begin{aligned} 7x' + 7y' + 2x &= 3e^t \\ x' + 3y' + y &= 3 \end{aligned}$$

[13]

6. Find a Fourier series for the following function

$$f(x) = \begin{cases} 1 & 0 \leq x \leq \pi \\ \cos x & \pi < x \leq 2\pi \end{cases} \quad ; \qquad f(x) = f(x + 2\pi)$$

[15]

7. Obtain the half-range Fourier **sine** series for the following function:

$$f(x) = 3 - x, \qquad 0 \leq x \leq 3 \quad \text{and} \quad f(x) = f(x + 6)$$

[6]

THE END