University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **MAT3A01 / MAT01A3**
REAL ANALYSIS

CAMPUS **APK**

EXAM **JUNE 2020**

Date 12/06/2020 Session 00:01 – 11:30

Assessor Dr G Braatvedt

External Moderator Dr L Lindeboom

Duration 2.25 Hours 40 Marks

Surname and initials:

Student number:

Tel No.:

INSTRUCTIONS:

1. The paper consists of **8** printed pages, **excluding** the front page.
2. Read the questions carefully and answer all questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Questions are to be answered on the question paper in the space provided. Please indicate when the blank side of a page is used.
5. **Good luck - write well :-)**

Question 1 [5]

Establish the convergence or divergence of the following sequences in full detail:

(a) $x_n = \left(1 - \frac{1}{2n}\right)^n$ (3)

(b) $y_n = \sqrt{n}$ (2)

Question 2 [6]

Prove or disprove that (z_n) is Cauchy if

$$z_n = \frac{2\sqrt{n}}{\sqrt{n}-1}.$$

(a) Using the definition of Cauchy. (3)

(b) Using any other technique. (3)

Question 3 [4]

Consider the function $f(x) = \frac{1}{x^2-1}$.
Show, using the definition of continuity, that f is continuous on $(-1, 1)$.

Question 4 [6]

Let $g(x) = \frac{1}{\sqrt{x}-1}$.

(a) Establish whether or not g is uniformly continuous on $[0,1)$ using the *Continuous Extension Theorem.* (3)

(b) Establish whether or not g is uniformly continuous on $[2,3]$. (3)

Question 5 [6]

Consider the function $h(x) = \frac{1}{x+3}$.

(a) Determine whether or not h is Lipschitz on $[-1, \infty)$. (3)

(b) Show that h is not uniformly continuous on $(-3, \infty)$, by making use of the *Nonuniform Continuity Criterion*. (3)

Question 6 [4]

Give an example of the following, if such an example exists (if no such example exists, explain why):

(a) A divergent sequence, with a large number K of convergent subsequences. (2)

(b) A function that is discontinuous on the whole of $\mathbb{R}$. (2)

Question 7 [4]

True or false (if true: give a short motivation; if false, give a counterexample):

(a) If (x_n) is unbounded and $x_n \leq y_n$ for all $n \in \mathbb{N}$, then (y_n) is divergent. (2)

(b) If f is continuous on $A \subseteq \mathbb{R}$ then f is uniformly continuous on A. (2)

Question 8 [5]

Let $f : A \to \mathbb{R}$. Assume that f has an absolute maximum and absolute minimum on each $\emptyset \neq B \subseteq A$.

(a) Prove or disprove that f is continuous on A. (2)

(b) Suppose, additionally, that $f(A)$ has a cluster point. Prove or disprove that f is continuous on A. (3)