University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MAT3A20 / MAT02A3

DISCRETE MATHEMATICS AND LOGIC

ONLINE SUPPLEMENTARY EXAM

JULY 2020

Examiner: Dr. F. Schulz
Moderator: Prof. W. Conradie (Wits)
Availability: 00:00 to 12:30 **35** marks

INSTRUCTIONS:

1. The paper consists of **3** pages, **including** the front page.
2. Answer all **nine** questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Read the questions carefully.
5. Questions are to be answered neatly on your own pieces of paper. Please ensure that your details (initials, surname and student number) are clearly written on the first page.
6. Please scan your answers and submit it as a **single** and **neat** pdf document before the deadline which is 12:30.
7. Non-programmable calculators are allowed.
8. This is an open book test.
9. Good luck!

Question 1 [3]

Let A, B and C be sets. Prove that

$$(A \cap B) - C \subseteq B - C.$$

Moreover, show by example that the set containment above can be strict.

Question 2 [6]

Let $\mathscr{G}$ be the set of all (undirected) graphs with vertex set $\{v_1, v_2, \ldots, v_n\}$, $n \geq 3$. Note that two graphs G_1 and G_2 in $\mathscr{G}$ are equal if and only if $E(G_1) = E(G_2)$. For any $G, H \in \mathscr{G}$ we write $H \subseteq^* G$ to indicate that H is a subgraph of G.

(a) Prove that $\subseteq^*$ is a partial order on $\mathscr{G}$. (3)

(b) Is $\subseteq^*$ a linear order? Motivate your answer. (1)

(c) Is $(\subseteq^*)^{\text{sym}}$ an equivalence relation? Justify your answer clearly. (2)

Question 3 [5]

(a) Determine whether or not $\neg p \vee (q \wedge r) \models \neg(r \to q)$ by making use of a truth table. (3)

(b) A set $\mathcal{S}$ of propositional formulas is said to be *satisfiable* if and only if there is some assignment of truth values to the propositional variables that satisfies all the formulas belonging to $\mathcal{S}$. Is $\{p \leftrightarrow \neg r, \neg p \vee (q \wedge r), \neg(r \to q)\}$ satisfiable? Motivate. (2)

Question 4 [3]

Make use of known equivalences to construct a DNF equivalent to the *negation* of the following formula:

$$p \leftrightarrow \neg((p \wedge q) \to r).$$

Question 5 [3]

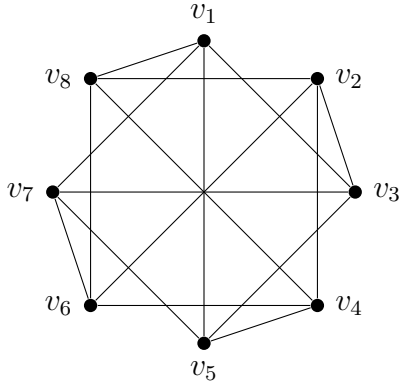
Show that if we take $n+1$ elements from the set $\{1, 2, \ldots, 2n-1, 2n\}$, then some pair of numbers will have no factors other than 1 in common.

Question 6 [5]

Solve the recurrence relation $a_n = 2\sqrt{3}a_{n-1} - 4a_{n-2}$ where $a_0 = 0$ and $a_1 = 1$.

Question 7 [4]

Consider the graph G depicted below and answer the questions that follow:



(a) Does G contain an Eulerian trail? Justify your answer clearly. (1)

(b) Is G bipartite? If so, give the two partite sets; if not, explain why not. (1)

(c) Determine whether or not the graph G is planar. If yes, draw it as a plane graph. If not, use Kuratowski's Theorem to prove that it is not. (2)

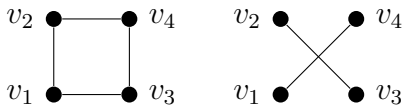
Question 8 [3]

(a) Prove that a graph G is a tree if and only if it has a vertex induced subgraph which is a spanning tree of G. (2)

(b) Can "vertex induced" be replaced by "edge induced" in part (a)? Motivate your answer. (1)

Question 9 [3]

The complement $\overline{G}$ of a graph G is a graph with the same vertex set as G such that for any two distinct vertices $u, v \in V(\overline{G}) = V(G)$, $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$. For instance, the following two graphs are the complements of each other:



Show that if T is a tree containing at least one vertex of degree 2, then $\overline{T}$ is not Eulerian.