UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

B-Tech Chemical and Metallurgy

MODULE: **MAT1AE3**

CAMPUS: **DFC**

ASSESSMENT: **JUNE EXAMINATION**

DATE: **09 JUNE 2020**   **DURATION 12:30 - 15:30**

ASSESSOR: **MR. MP SELOANE**

MOERATOR: **Prof. M KHUMALO**

TOTAL: 78 MARKS

INSTRUCTIONS:

ANSWER ALL QUESTIONS.

WRITE YOUR STUDENT NUMBER, SURNAME, AND INITIALS AT THE TOP OF EACH OF YOUR ANSWER SHEET(S).

WHEN YOU HAVE FINISHED WRITING, UPLOAD YOUR ANSWER SHEET(S) ON BLACKBOAD AS DIRECTED ON THE SAME PLATFORM.

Question 1 [18]

1.1 Solve the following DE

$$\frac{dy}{dx} - \frac{1}{x}y = -\frac{y^2}{x^2}$$ (6)

1.2 Show that the solution to the DE

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \text{ is } xC = y^2 + x^2 ,$$ (6)

where C is an arbitrary constant.

1.3 Test the following DE for exactness and then solve. (6)
$$(y \sin x + xy \cos x + 2xy^2)dx + (x \sin x + 2x^2y)dy = 0 .$$

Question 2 [3]

A certain virus is spreading through a city at a rate of

$$\frac{dI}{dt} = 3(t + 1)$$

where I is the number of infected people and t is measured in days. If the initial number of infected people is 10, find the general solution for the number of infected people. (3)

Question 3 [6]

Evaluate the following Laplace transforms:

3.1 $L\{sin^2 3t\}$ (3)

3.2 $L\left\{\int_0^t t \cosh t\right\}$ (3)

Question 4 [7]

Calculate the following inverse Laplace transforms:

4.1 $L^{-1}\left\{\frac{p}{p^2 + 4p + 5}\right\}$ (3)

4.2 $L^{-1}\left\{\frac{p\,e^{-2p}}{p^2 - 3p - 4}\right\}$ (4)

Question 5 [10]

Let $f(t) = \begin{cases} t & 0 \le t < 2 \\ t - 2 & 2 \le t < 3 \\ 0 & t \ge 3 \end{cases}$

  5.1 Sketch $f(t)$.   (3)

  5.2 Write $f(t)$ in Heaviside form.   (3)

  5.3 Find the Laplace transforms of $f(t)$ (4)

Question 6 [8]

Solve the following IVP using D-operator methods.   (8)

$$y'' + 2y' = 1 - 3x$$

Question 7 [7]

Solve for x only in the following system using D-operators.   (7)

$$(D + 1)x + (D + 1)y - 4e^{3t} = 0$$

$$3x + D^2y = e^{2t} - 2x$$

Question 8 [10]

  8.1 Estimate the root of $f(x) = e^{-x} - \log x$ with three-decimal convergence

     given $x = 1.6$ as the initial guess.   (5)

  8.2 If $\frac{dy}{dx} = e^x + \frac{y}{x}$, $y(1) = 1$, find the RK-4 approximation over the interval

     $1 \le x \le 1.2$ with step size $h = 0.2$.   (5)

Question 9 [8]

9.1 Solve the following system using the Gaussian elimination method.   (5)

$$x_1 - x_2 + 2x_3 = 1$$
$$x_1 + x_2 + x_3 = 3$$
$$2x_1 - x_3 = 2$$

9.2 Find the directional derivative of $f(x,y) = x^2 + 2xy + 2y^2$ at $P(1,3)$ in the direction of $v = \mathbf{i} + 4\mathbf{j}$.   (3)

******** END OF EXAM ******