UNIVERSITY OF JOHANNESBURG

UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	**MAT3A01 / MAT01A3** REAL ANALYSIS
CAMPUS	**APK**
EXAM	**JULY 2020**

DATE 13/07/2020 — Session 00:01 – 11:00

ASSESSOR — Dr G Braatvedt

EXTERNAL MODERATOR — Dr L Lindeboom

DURATION 2.25 HOURS — 40 MARKS

SURNAME AND INITIALS:

STUDENT NUMBER:

TEL NO.:

INSTRUCTIONS:

1. The paper consists of **8** printed pages, **excluding** the front page.
2. Read the questions carefully and answer all questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Questions are to be answered on the question paper in the space provided. Please indicate when the blank side of a page is used.
5. **Good luck - write well :-)**

Question 1 [5]

Establish the convergence or divergence of the following sequences in full detail:

(a) $x_n = 1 - \frac{(-1)^n}{2n}$ (2)

(b) $y_n = \frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{n^2}$ (3)

Question 2 [6]

Prove or disprove that (z_n) is Cauchy if

$$z_n = \frac{2n}{\sqrt{n}-1}.$$

(a) Using the definition of Cauchy. (3)

(b) Using any other technique. (3)

Question 3 [4]

Consider the function $f(x) = \frac{1}{x^2-4}$.
Show, using the definition of continuity, that f is continuous on $(-\infty, -2)$.

Question 4 [6]

Let $g(x) = \frac{x-1}{\sqrt{x}-1}$.

(a) Establish whether or not g is uniformly continuous on $[0, 1)$. (3)

(b) Establish whether or not g is uniformly continuous on $[2, \infty)$. (3)

Question 5 [6]

Consider the function $h(x) = \frac{x}{x+3}$.

(a) Determine whether or not h is uniformly continuous on $(-\infty, -3)$. (3)

(b) Determine whether or not h is uniformly continuous on $[-2, 4]$. (3)

Question 6 [4]

Give an example of the following, if such an example exists (if no such example exists, explain why):

(a) A divergent sequence, with no convergent subsequences. (2)

(b) An unbounded function with a countable range, that is not monotone. (2)

Question 7 [4]

True or false (if true: give a short motivation; if false, give a counterexample):

(a) If $0 \leq x_n \leq y_n$ for all $n \in \mathbb{N}$, and (y_n) is convergent, then (x_n) is convergent. (2)

(b) If $f : A \to \mathbb{R}$ is continuous on A then f has an absolute maximum on A. (2)

Question 8 [5]

Let $f : A \to \mathbb{R}$. Assume that f has an absolute maximum and absolute minimum on each $\emptyset \neq B \subseteq A$ and that $f(A)$ has a cluster point.
Prove or disprove that f is continuous on A.