APPLIED MATHEMATICS

Quantum Computing
APM8X16

Examination: 03/06/2020

Marks: 50
Assessor: Dr. G.J. Kemp
Moderator: Prof. Y. Hardy

Question 1 (15 marks)

This question concerns the Bloch sphere and the rotation matrices.

(a) Calculate the eigenvectors of the three Pauli matrices, X, Y, Z. Draw the Bloch sphere and indicate the positions of each eigenvector on the Bloch sphere. (6)

(b) Rotate the vector (6)

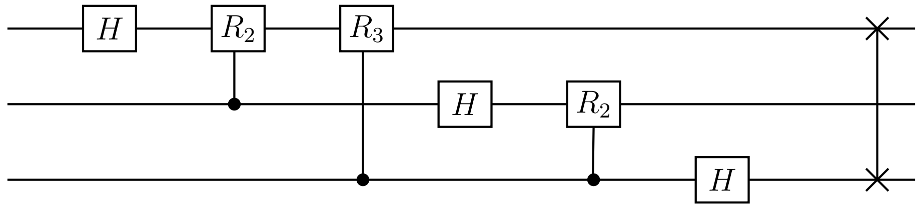
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

by an angle $\pi/2$ about the z-axis. Where is this new vector on the Bloch sphere?

(c) What is the net effect of the product $R_x(\pi/2)R_x(2\pi/3)$? (3)

Question 2 (10 marks)

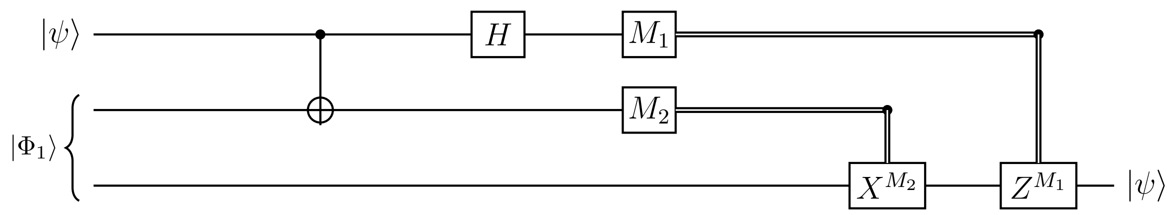
This question concerns the quantum Fourier transform.



Calculate the output state for the input state $|j_1 j_2 j_3\rangle = |101\rangle$.

Question 3 (10 marks)

The following circuit implements the quantum teleportation algorithm.



The top two registers belong to Alice and the bottom one belongs to Bob. Alice and Bob share the entangled state $|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice wants to transmit the qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob using the above circuit.

(a) Calculate the overall state just before Alice performs the measurements M_1 and M_2. (7)

(b) Say Alice performs a measurement on her two qubits and obtains the result $M_1 = 1, M_2 = 0$. What is the probability of obtaining this result? Describe the transformation that Bob must now apply to his qubit to obtain the desired state ψ. (3)

Question 4 (15 marks)

This question concerns the Phase Estimation algorithm.

(a) Let the final state of the Phase Estimation algorithm be the superposition (8)

$$|\psi_f\rangle = \sum_{l=0}^{2^t-1} \alpha_{\varphi,l} \, |l\rangle \, |u\rangle \, ,$$

where $|u\rangle$ is the eigenvector of a unitary U, and φ is the unknown in the phase of the corresponding eigenvalue $e^{2\pi i\varphi}$. The integer l is what one would measure at the end of the algorithm. The probability for the outcome l is $\mathbb{P}(l)$. Calculate

$$\sum_{l=0}^{2^t-1} \mathbb{P}(l).$$

Is this answer to be expected?

(b) Consider the following 2×2 unitary matrix (7)

$$U = \begin{pmatrix} -\frac{i}{2} + \frac{1}{\sqrt{2}} & -\frac{i}{2} \\ -\frac{i}{2} & \frac{i}{2} + \frac{1}{\sqrt{2}} \end{pmatrix}$$

The value of φ for one of its eigenvalues is $7/8$. Say you design your Phase Estimation circuit with $t = 5$ qubits in the top half. Which l value is the most likely outcome upon measurement? What is the probability $\mathbb{P}(l)$ for this l value? Would it be useful in this case to increase t in your quantum circuit? Explain.

(c) As a bonus question, what is the smallest value for t you can use in your circuit to determine the phase in 4(b) exactly? (2)

END OF QUESTION PAPER