APPLIED MATHEMATICS

Differential Equations A
APM8X10

Supplementary Online Assessment: 15/07/2020

Maximun Duration: 2 hours 30 minutes **Marks:** 100
Assessor: Dr F. Chirove
Moderator: Dr F. J. Mhlanga, University of Limpopo

Instructions:

1. Carefully read and follow the instructions of each question.
2. Answer all questions and submit them in the order they are given.
3. All calculations must be shown.
4. Calculators are permitted and you need one in this case.
5. Download the pdf and complete at home.
6. Upload your answer script on blackboard before 1100hrs after which you will not have access to submit.
7. Merge your work into a single PDF document and make sure solutions are in numerical order.
8. Click on the bold exercise link (**APM8X10 Final assessment**) and attach your solutions as a single PDF document. You have a SINGLE ATTEMPT ONLY so make sure that you upload the final document.
9. At the bottom of your last solution page, please write and sign the following : **Integrity Declaration - I declare that the work in this final assessment is completely my own work. No part of this assessment was a result of collaboration with others students within and without my class.**

Question 1 (25 marks)

(a) Consider the second order initial-value problem (IVP) $y'' + P(x)y' + Q(x)y = f(x),\ y(x_0) = y_0,\ y'(x_0) = y_1$ on the interval $(0, \infty)$. Let $y_(x) = \int_{x_0}^{x} G(t,x)f(t)dt$ where $G(t,x)$ is the Green's function of the IVP. If $y_h(x)$ is a solution of the associated homogeneous differential equation (DE) with nonhomogeneous initial conditions and $y_p(x)$ is a solution of the nonhomogeneous differential equation (DE) with homogeneous initial conditions, prove that $y(x) = y_h(x) + y_p(x)$ is a solution of the IVP. (7)

(b) Solve the initial-value problem using Green's functions (18)

$$y'' + 4y = f(x), \quad y(0) = 1, y'(0) = -2,$$

where

$$f(x) = \begin{cases} 0, & x < 0, \\ \sin 2x, & 0 \le x \le 2\pi, \\ 0, & x > 2\pi. \end{cases}$$

Question 2 (25 marks)

(a) Determine all the ordinary and singular points of the Bessel's equation (7)

$$x^2y'' + xy' = (x^2 - \frac{1}{49})y = 0.$$

For all singular points, classify them as either regular or irregular.

(b) Solve the Bessel's equation in part (a) about $x = 0$. (18)

Question 3 (25 marks)

(a) Solve the **nonhomogeneous** system (17)

$$\vec{\boldsymbol{y}}\,' = \begin{bmatrix} 2 & -1 \\ 5 & 0 \end{bmatrix} \vec{\boldsymbol{y}} + \begin{bmatrix} -3e^t \\ -e^t \end{bmatrix}$$

using the method of variation of parameters.

(b) Let (8)

$$\vec{\boldsymbol{y}}_{\mathbf{1}}(t) = \begin{bmatrix} t^2 - 2t \\ 2t \end{bmatrix}, \quad \vec{\boldsymbol{y}}_{\mathbf{2}}(t) = \begin{bmatrix} t-1 \\ 1 \end{bmatrix}.$$

Compute the Wronskian to determine whether the functions $\vec{\boldsymbol{y}}_{\mathbf{1}}(t)$ and $\vec{\boldsymbol{y}}_{\mathbf{2}}(t)$ are linearly independent on $(0, \infty)$. State giving reasons whether or not the functions form the fundamental set of solutions. If so, give the general solution of the system.

Question 4 (25 marks)

(a) Consider the system of equations (14)

$$\frac{dx}{dt} = x\left(1 - \frac{x}{3} - y\right)$$

$$\frac{dy}{dt} = y\left(1 - \frac{y}{4} - x\right).$$

Find all critical points and determine their stability.

(b) Consider the nonlinear ordinary differential equation with a parameter k (11)

$$\frac{dx}{dt} = x(x-1) - k.$$

Determine the bifurcation points and classify them. Draw a bifurcation diagram showing the direction of evolution of solutions.

END OF QUESTION PAPER