APPLIED MATHEMATICS

Differential Equations A
APM8X10

Final Online Assesment: 05/06/2020

Maximun Duration: 2 hours 30 minutes **Marks:** 100
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Moderator: Dr F. J. Mhlanga, University of Limpopo

Instructions:

1. Carefully read and follow the instructions of each question.
2. Answer all questions and submit them in the order they are given.
3. All calculations must be shown.
4. Calculators are permitted and you need one in this case.
5. Download the pdf and complete at home.
6. Upload your answer script on blackboard before 1100hrs after which you will not have access to submit.
7. Merge your work into a single PDF document and make sure solutions are in numerical order.
8. Click on the bold exercise link (**APM8X10 Final assessment**) and attach your solutions as a single PDF document. You have a SINGLE ATTEMPT ONLY so make sure that you upload the final document.
9. At the bottom of your last solution page, please write and sign the following : **Integrity Declaration - I declare that the work in this final assessment is completely my own work. No part of this assessment was a result of collaboration with others students within and without my class.**

Question 1 (26 marks)

(a) Sovle the IVP $169x^2y'' + 91xy' + 9y = 0,\ y(1) = y_0,\quad y'(1) = y_1$ on the interval $(0, \infty)$. (10)

(b) For what values of y_0 and y_1 does the initial-value problem have a solution? (2)

(c) Consider the boundary-balue problem (4)

$$\begin{aligned} L[y] &= f(x), \quad L[y] = y'' + P(x)y' + Q(x)y, \\ A_1y(a) + B_1y(a) &= 0, \\ A_2y(b) + B_2y(b) &= 0, \end{aligned}$$

where $P(x)$, $Q(x)$ and $f(x)$ are continuous on $[a, b]$. Prove that a function $u(x)$ satisfies $L[u] = 0$ and $A_1u(a) + B_1u(a) = 0$ if and only if $u = \lambda y_1$ for some $\lambda \in \Re$ and y_1 a unique solution of the initial value problem of $L[y] = 0,\ y(a) = B_1,\ \ y'(a) = -A_1$.

(d) Find the Green's function for the boundary value problem (10)

$$y'' + 4y = f(x), \quad y(0) = 0,\ \ y'(1) = 0.$$

Question 2 (21 marks)

(a) Determine all the ordinary and singular points of the equation (4)

$$(x + 4)y'' - (9 - x)y' + y = 0$$

and calculate the minimun radius of convergence about the point $x = 0$.

(b) Use limits to determine if the singular point(s) are regular or irregular. (5)

(c) Find the power series solutions of the differential equation (12)

$$(x + 4)y'' - (9 - x)y' + y = 0$$

about the point $x = 0$ computing constants upto c_4.

Question 3 (21 marks)

(a) Write the system in matrix form (3)

$$\begin{aligned} \frac{dx}{dt} &= -3x + 4y + e^{-t}\sin 2t, \\ \frac{dy}{dt} &= 5x + 9z + 4e^{-t}\cos 2t, \\ \frac{dz}{dt} &= -y + 4z - e^{-t}. \end{aligned}$$

(b) Write the system without the use of matrices (3)

$$\frac{dX}{dt} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & -2 \\ 5 & -2 & 6 \end{bmatrix} X + \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} e^{-2t} - \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} t.$$

(c) Verify that $y(t) = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} e^{13t}$ is a solution to the differential equation (5)

$$y'(t) = \begin{bmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{bmatrix} y(t).$$

(d) Solve the system (10)

$$\frac{dx}{dt} = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix} x \text{ with } x(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}.$$

Question 4 (32 marks)

(a) Consider the system of equations (14)

$$\frac{dx}{dt} = x\,(2 - 2x - y)$$

$$\frac{dy}{dt} = y\,(3 - 2y - x)\,,$$

taking $(x, y) > 0$. Find all critical points and determine their stability.

(b) Consider the nonlinear ordinary differential equation with a parameter k (18)

$$\frac{dx}{dt} = kx + x^2 + x^3.$$

Determine the bifurcation points and classify them. Draw a bifurcation diagram showing the direction of evolution of solutions.

END OF QUESTION PAPER