Question Number	Marks Awarded
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL:	

Assessor: Prof F. Nyabadza
Moderator: Dr R. Ouifki
Duration: Online
Marks: 100

APPLIED MATHEMATICS

Dynamical Systems A
APM8X01
Examination: 11/06/2020

Name: ______________________ **Student Number:** ______________________

Instructions:

1. Check that this question paper consists of 2 pages in total.
2. All calculations must be shown.
3. Pocket calculators are permitted.

Question 1 [6 marks]
Find the fixed orbits of
$$f(x) = -x^5$$

Question 2 [6 marks]
Perform complete orbit analysis of
$$f(x) = x - x^2$$

Question 3 [17 marks]
Let $g(x) = |x - a|$

(a) Compute $g^2(x)$ and $g^3(x)$. (6)

(b) Find all the fixed points of $g(x)$. (6)

(c) How many periodic orbits does $g(x)$ have if $a = 1$, if any? (5)

Question 4 [15 marks]
Given that
$$x_{n+1} = \alpha x_n + \beta \qquad (*)$$

(a) Find the steady state(s) and state the condition under which the steady state exists. (5)

(b) Show that $y_n = x_n - x^*$ transforms $(*)$ into a homogeneous linear equation. (5)

(c) Use the result in (b) to find the general solution of $(*)$ (5)

Question 5 [16 marks]
A population model is given by
$$\frac{dN}{dt} = f(N) = \alpha N e^{-\beta N}, \qquad \alpha, \beta > 0.$$

(a) What could be the possible interpretation of α and β,, justifying your answers? (4)

(b) Sketch $f(N)$ against N. (4)

(c) Find the steady states and determine their stabilities. (8)

Question 6 [10 marks]
Given that
$$\frac{dx}{dt} = x(r - e^x).$$

(a) Find the steady states. (5)

(b) Draw the bifurcation diagram. (3)

(c) What type of bifurcation is exhibited? (2)

Question 7 [14 marks]

A model with 3 phenotypes has the probability of getting the dominant alleles in the next generation given by the following map

$$f_{n+1} = f(P_n) = \frac{(\alpha-\beta)p_n^2 + \beta p_n}{(\alpha - 2\beta + \gamma)p_n^2 - 2(\alpha-\beta)p_n + \gamma}.$$

(a) Show that the map has three steady states. (8)

(b) Using cobwebbing to determine the stabilities of the steady states for the case $\beta > \alpha,\ \gamma$. (6)

Question 8 [16 marks]

Consider the Nicholson and Bailey model, for a host-parasitisoid interaction given by

$$H_{n+1} = kH_n e^{-aP_n}$$
$$P_{n+1} = cH_n\left(1 - e^{-aP_n}\right)$$

(a) Find the steady states of the system of equations. (6)

(b) Find the Jacobian matrix at the non-trivial steady state. (6)

(c) Use the Jurry conditions to prove the stability of the non- trivial steady state. (4)

END OF QUESTION PAPER