UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE MECHANICAL ENGINEERING
SUBJECT	:	**THERMOMACHINES 4A**
CODE	:	**TRM4A11**
DATE	:	SUPPLEMENTARY EXAMINATION JULY 2019
DURATION	:	(1-PAPER) 3 HOURS
WEIGHT	:	50 : 50
TOTAL MARKS	:	102, MARKED OUT OF 100

EXAMINER	:	Dr CR BESTER
MODERATOR	:	Mr GH JANSEN VAN RENSBURG
NUMBER OF PAGES	:	2 PAGES INSTRUCTIONS 4 PAGES QUESTIONS WITH SOLUTIONS 4 PAGES ANNEXURE (FORMULAE) 1 PAGE FUEL CHART

INSTRUCTIONS	:	SEE NEXT PAGE
REQUIREMENTS	:	NONE

INSTRUCTIONS TO CANDIDATES:

- FORMULA SHEETS AND FUEL CHART ATTACHED
- NO BOOKS, LECTURE NOTES, STUDY-, HOMEWORK- OR TUTORIAL MATERIAL ALLOWED
- UJ APPROVED CALCULATORS ALLOWED
- NO ANSWERS IN PENCIL OR RED INK WILL BE ACCEPTED
- ANSWER ALL 5 QUESTIONS IN ENGLISH
- SMOKING IS PROHIBITED DURING THE DURATION OF THE EXAM
- NOTE THE USE OF THE DECIMAL COMMA

QUESTION 1 **(21 marks)**

An intricate crank mechanism is used to create a spark ignition engine with an expansion stroke that is longer than the compression stroke. The aim of the cycle is to obtain complete expansion of the working substance to atmospheric pressure, thereby increasing work output.

The *P-v* diagramme of a spark ignition engine cycle with complete expansion is shown in figure 1. Expansion occurs from state 3 to state 4, which is at the same pressure as state 1. Heat rejection occurs from state 4 to state 1, at constant pressure.

The cycle processes are as follows

1 – 2: Isentropic compression
2 – 3: Constant volume heat input
3 – 4: Isentropic expansion
4 – 1: Constant pressure heat rejection

Specific volume ratios of the cycle are as follows

$\alpha = v_4/v_3$
$\beta = v_1/v_4$

Express the thermal efficiency of the cycle in terms of α, β and the ratio of specific heat capacities γ.

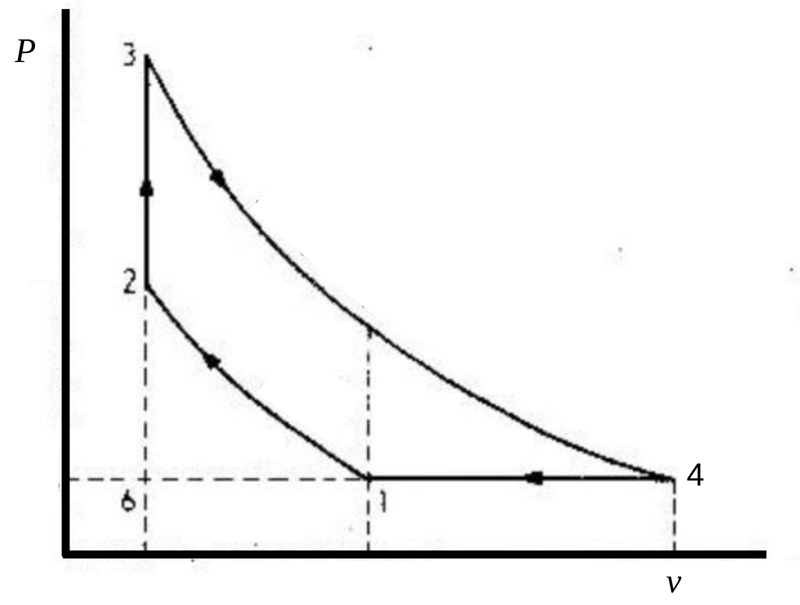


Figure 1: *P-v* diagramme of complete expansion spark ignition cycle [Obert* p. 175 Figure 6-7]

* Obert, E.F., "Internal Combustion Engines and Air Pollution," Intext Harper & Row, New York, 1973

QUESTION 2 (15 marks)

Consider ideal air-standard Otto- and Diesel cycles. The Otto cycle has a compression ratio of 10:1. Its temperature after combustion is three times its temperature before combustion, i.e. $T_3/T_2 = 3$. For equal heat inputs and equal thermal efficiencies of the two cycles, determine the compression ratio of the Diesel cycle. The intake temperature is 288,15 K for both cycles. Working substance properties are as follows:

R: 287,1 J/kgK; γ: 1,35; C_V: 820,29 J/kgK; C_P: 1107,4 J/kgK

QUESTION 3 (12 marks)

Consider the schematic of an ideal turbopropeller engine shown in figure 2.

Derive expressions for pressures P_2 to P_5 and temperatures T_2 to T_5 in terms of the cycle pressure ratio r_P, working substance ratio of specific heats γ, maximum-to-minimum temperature ratio t and entry conditions P_1 and T_1.

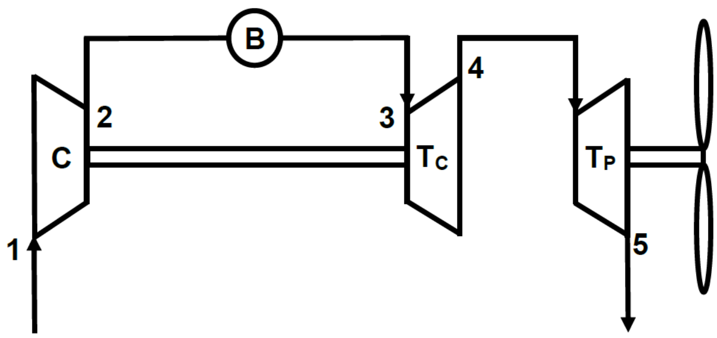


Figure 2: Ideal turbopropeller engine

QUESTION 4 (34 marks)

A single-shaft turbojet is operating under the following conditions:

Altitude	11 000 m
Flight Mach number	0,85
Compressor pressure ratio	12
Isentropic efficiencies	
Intake diffuser	100%
Compressor	87%
Turbine	89%
Nozzle	100%
Combustion efficiency	98%
Combustion chamber pressure drop	4% of P_{02}
Mechanical efficiency of turbine-compressor spool	99%
Turbine inlet temperature	1400 K

An engine schematic with a *T-s* diagramme is shown in figure 3.

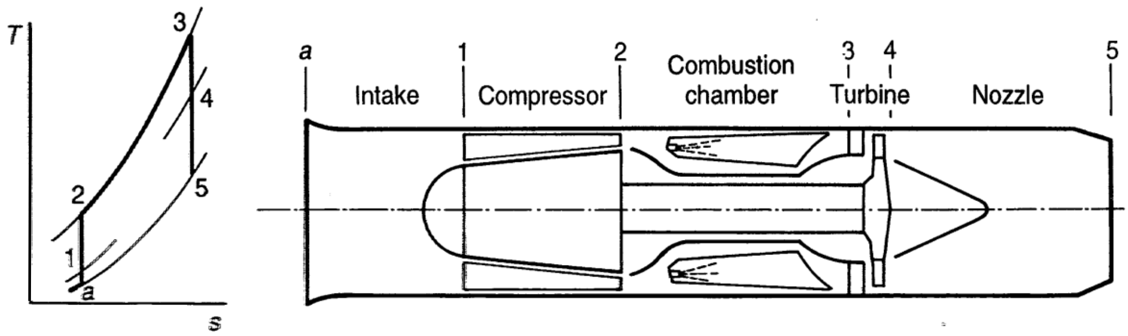


Figure 3: Turbojet schematic [CRSS p. 102 figure 3.5]

Calculate

(i) The ambient temperature, -pressure and -density at an ASL of 11 000 m
(ii) Aircraft speed
(iii) All the pressures and temperatures of the cycle
(iv) The net specific thrust and propulsion efficiency for a convergent nozzle
(v) The net specific thrust and propulsion efficiency for a convergent-divergent nozzle

The working substance properties are as follows:

$\gamma_a = 1,4$; C_{Pa} = 1 005 J/kgK; $\gamma_g = 4/3$; C_{Pg} = 1 148 J/kgK

QUESTION 5 (20 marks)

Using the following equation, determine seven important π-numbers of incompressible flow turbomachines. From the parameters below select the primary parameters. Determine the dimensions of all the parameters and apply Dimensional Analysis to obtain the required dimensionless ratios.

$$\pi = \pi\,(\rho,\quad N,\quad D,\quad P,\quad \tau,\quad F,\quad \dot{W},\quad \varepsilon,\quad \mu,\quad Q)$$

Annexure: Formula sheets

$$\eta = \frac{T'_{01} - T_a}{T_{01} - T_a}$$

$$c = \sqrt{\gamma RT}$$

$$\eta = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$M_a = c_a / c$$

$$\eta = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$P_{03} = P_{02}\left(1 - \frac{\Delta P_b}{P_{02}} - \frac{\Delta P_{ha}}{P_{02}}\right)$$

$$\eta = \frac{(FA)_{theor}}{(FA)_{actual}}$$

$$P_a = P_{04}\left(1 - \frac{\Delta P_{hg}}{P_{04}}\right)$$

$$\eta = \frac{T_{04} - T_5}{T_{04} - T'_5}$$

$$w = C_p(T_{01} - T_{02})$$

$$\eta = \frac{T_{04} - T_c}{T_{04} - T'_c}$$

$$w = C_p(T_{03} - T_{04})$$

$$\eta = \frac{P_{01} - P_a}{P_{0a} - P_a}$$

$$C_p - C_v = R$$

$$\eta = -\frac{w_C}{w_T} = -\frac{\dot{w}_C}{\dot{w}_T}$$

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

$$\eta = \frac{w_{net}}{q_{in}}$$

$$\frac{n-1}{n} = \frac{\gamma - 1}{\eta_p \gamma}$$

$$w_T = -\frac{w_C}{\eta} + \frac{w_{net}}{\eta}$$

$$\frac{n-1}{n} = \eta_p \frac{\gamma - 1}{\gamma}$$

$$\alpha = 1$$

$$\phi = (AF)_{stoich} / (AF)_{act}$$

$$\lambda = 1/\phi$$

$$SFC = 3600\dot{m}_f / \dot{W}$$

$$SFC = \dot{m}_f / \dot{W}$$

$$\eta = ISFC / BSFC$$

$$\eta = \frac{m_a}{\rho_a V_d}$$

$$\eta = \frac{60\dot{m}_a}{\alpha \rho_a V_d N} = \frac{60 n \dot{m}_a}{\rho_a V_d N}$$

$$W_{61} = P_{in}(V_1 - V_6) = P_{in} V_d$$

$$W_{56} = P_{ex}(V_6 - V_5) = -P_{ex} V_d$$

$$(W_{pump})_{net} = (P_{in} - P_{ex}) V_d$$

$$l_{max} < d_v / 4$$

$$C_{Dv} = A_{act} / A_{pass}$$

$$A_{pass} = \pi d_v l$$

$$A_i = \frac{\pi}{4} d_v^2 = C B^2 \frac{\bar{\dot{y}}_{max}}{c_i}$$

$$C \approx 1{,}3$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$T_1 v_1^{\gamma - 1} = T_2 v_2^{\gamma - 1}$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$\dot{Q} = \dot{m} C_v \Delta T$$

Formulae (continued)

$$\dot{Q} = \dot{m}(C_p + C_v)\Delta T$$

$$m_f q_{in} = (m_a + m_f)C_v \Delta T$$

$$m_f q_{in} = (m_a + m_f)C_p \Delta T$$

$$(m_a + m_f)C_{Pg}(T_{03} - T_{02}) = \eta_b m_f Q_{LHV}$$

$$dq = du + dw$$

$$dq = dh - vdP$$

$$dw = Pdv$$

$$q_{in} + q_{out} = w_{net}$$

$$t = T_3/T_1$$

$$F = \dot{m}(c_j - c_a) + A_j(P_j - P_a)$$

$$F = \dot{m}(c_j - c_a)$$

$$SFC = \frac{3600(FA)}{F_s}$$

$$T(h) = T_{SL} - \lambda h$$

$$P(h) = P_{SL}\left(1 - \frac{\lambda}{T_{SL}}h\right)^{g/(\lambda R)}$$

$$\rho(h) = \rho_{SL}\left(1 - \frac{\lambda}{T_{SL}}h\right)^{\left(\frac{g}{\lambda R} - 1\right)}$$

$$C_p = 1\ 005\ J/kgK$$

$$C_p = 1\ 148\ J/kgK$$

$$R = 287{,}1\ J/kgK$$

$$P = \rho RT$$

$$C_p T_0 = C_p T + c^2/2$$

$$\frac{T_0}{T} = \left(\frac{P_0}{P}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(n-1)/n}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}}\right)^{(n-1)/n}$$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_{04}}{T_{03}} = \left(\frac{P_{04}}{P_{03}}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_c}{T_{04}} = \left(\frac{P_c}{P_{04}}\right)^{(\gamma-1)/\gamma}$$

$$T_c = \frac{2}{\gamma + 1}T_0$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$\gamma = 1{,}4$$

$$\gamma = 1{,}333$$

$$\lambda = -0{,}986\ °C/km$$

$$\lambda = 0\ °C/km$$

$$\lambda = 6{,}5\ °C/km$$

$$g = 9{,}81\ m/s^2$$

$$\dot{m}_{fuel} = (FA)\dot{m}_a$$

$$\dot{m} = \frac{\dot{W}}{w}$$

$$\dot{m} = \rho c A$$

$$\dot{m} = F/F_s$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

Formulae (continued)

$$F_D = \dot{m}c_a = (\dot{m}_c + \dot{m}_h)c_a$$

$$F_N = F_G - F_D$$

$$w_{T\,LP} = -\frac{w_F}{\eta}$$

$$T_{05} - T_{06} = (1+B)\frac{C_{pa}}{\eta_m C_{pg}}(T_{02} - T_{01})$$

$$\mathrm{T_{SL}} = 288{,}15\ K$$

$$P_{SL} = 101{,}325\ kPa$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \dot{m}_c / \dot{m}_h$$

$$\dot{m}_h = \dot{m}/(1+B)$$

$$\dot{m}_c = \dot{m}B/(1+B)$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$F_D = \dot{m}c_a = (\dot{m}_c + \dot{m}_h)c_a$$

$$\dot{W}_{T\,LP} = \dot{m}_h C_{pg}(T_{05} - T_{06})$$

$$\dot{W}_{T\,LP} = -\dot{W}_F / \eta_m$$

$$TP = \eta_{pr} SP + Fc_a$$

$$c = v_3 / v_x$$

$$EP = \frac{TP}{\eta_{pr}} = SP + \frac{Fc_a}{\eta_{pr}}$$

$$c = v_3 / v_2$$

$$EP_{take\,off} = \frac{TP}{\eta_{pr}} = SP + \frac{F}{8{,}5}$$

$$\alpha = P_x / P_2$$

$$\eta_{T\ Diesel} > \eta_{T\ Dual} > \eta_{T\ Otto}$$

$$s = a\cos\theta + \sqrt{r^2 - a^2\sin^2\theta}$$

$$y = (r+a) - (a\cos\theta + r\cos\phi)$$

$$R = r/a$$

$$\dot{y} = a\dot{\theta}\sin\theta\left(1 + \frac{\cos\theta}{\sqrt{R^2 - \sin^2\theta}}\right)$$

$$\dot{y} \approx a\Omega\left(\sin\theta + \frac{\sin 2\theta}{2R}\right)$$

$$\bar{\dot{y}} = 2S\,\frac{N}{60}$$

$$\frac{\dot{y}}{\bar{\dot{y}}} \approx \frac{\pi}{2}\left(\sin\theta + \frac{\sin 2\theta}{2R}\right)$$

$$V_d = V_{BDC} - V_{TDC}$$

$$V_d = \pi B^2 S / 4$$

$$V_d = N_c \pi B^2 S / 4$$

$$r_V = (V_c + V_d)/V_c$$

$$W = \int F\ dy$$

$$W = \int P\ dV$$

$$w = \int P\ dv$$

$$F = PA_p$$

$$A_p = \pi B^2 / 4$$

$$w_{net} = w_{gross} + w_{pump}$$

$$w_b = w_i - w_f$$

$$\eta_m = W_b / W_i$$

$$MEP = W / V_d$$

$$(IMEP)_{net} = (IMEP)_{gross} + PMEP$$

$$BMEP = (IMEP)_{net} - FMEP$$

$$BMEP = \eta_m (IMEP)_{net}$$

$$FMEP = (1 - \eta_m)(IMEP)_{net}$$

$$BMEP = 2\pi T_b / \alpha V_d$$

Formulae (continued)

$$\dot{W} = 2\pi NT/60$$

$$\dot{W}_{b\,engine} = (BMEP)V_d N\alpha/60$$

$$BMEP = 2\pi T_b/\alpha V_d$$

α = ½

$$m_f q_{in} = (m_a + m_f)C_v \Delta T$$

$$(AF)_{stoich} = 4{,}319\frac{32n+8m}{12n+1{,}008m}$$

$$\eta = 1 - \frac{1}{r_T}$$

$$\eta = 1 - \frac{1}{r_P^{(\gamma-1)/\gamma}}$$

$$\eta = 1 - \frac{r_T}{t}$$

$$\eta = \frac{2t - r_T + 1 - 2t/\sqrt{r_T}}{2t - r_T - t/\sqrt{r_T}}$$

$$w_{\text{net}}/C_P T_1 = 2t - r_T + 1 - 2t/\sqrt{r_T}$$

$$SFC = \frac{3600(FA)_{\text{actual}}}{w_{TP}}$$

$$\eta_T = \frac{w_{TP}}{Q_{HV}(FA)_{\text{actual}}}$$

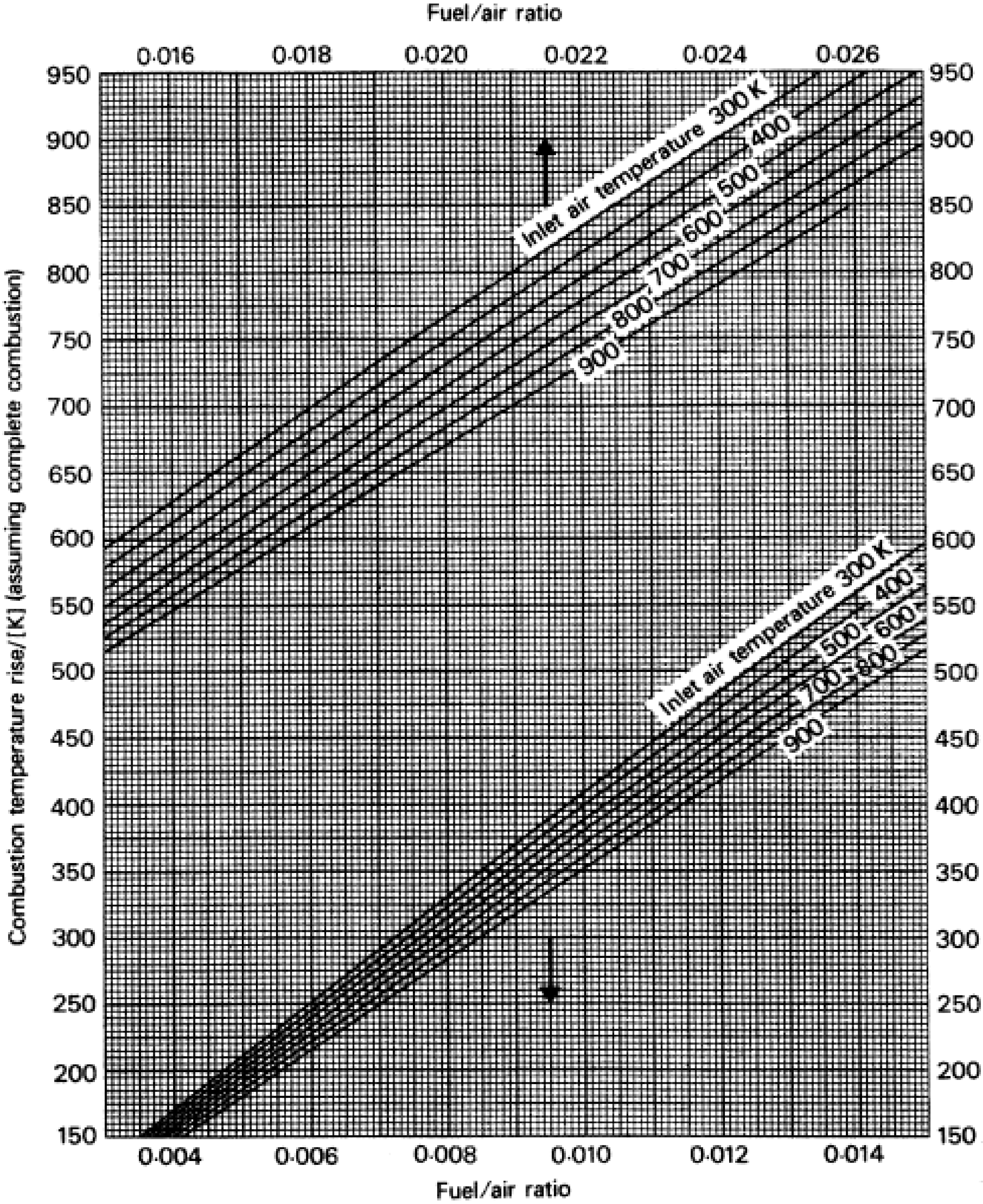
$$\eta = 1 - \frac{1}{r_V^{\gamma-1}}$$

$$W_{\text{in}} = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$W_{\text{out}} = \frac{P_4V_4 - P_3V_3}{1-\gamma}$$

$$C_nH_m + \left(n + \frac{m}{4}\right)O_2 + 3{,}76\left(n + \frac{m}{4}\right)N_2 \rightarrow nCO_2 + \left(\frac{m}{2}\right)H_2O + 3{,}76\left(n + \frac{m}{4}\right)N_2$$

$$\eta = \left(\frac{2}{1 + (c_j/c_a)}\right)100\%$$


Fuel/air ratio
0·016
0·018
0·020
0·022
0·024
0·026
Combustion temperature rise/[K] (assuming complete combustion)
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
Inlet air temperature 300 K
400
500
600
700
800
900
0·004
0·006
0·008
0·010
0·012
0·014
Fuel/air ratio