UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE MECHANICAL ENGINEERING
SUBJECT	:	**THERMOMACHINES 4A**
CODE	:	**TRM4A11**
DATE	:	EXAMINATION 25 MAY 2019
DURATION	:	(1-PAPER) 3 HOURS
WEIGHT	:	50 : 50
TOTAL MARKS	:	107, MARKED OUT OF 100

EXAMINER	:	Dr CR BESTER
MODERATOR	:	Mr GH JANSEN VAN RENSBURG
NUMBER OF PAGES	:	2 PAGES INSTRUCTIONS 4 PAGES QUESTIONS WITH SOLUTIONS 4 PAGES ANNEXURE (FORMULAE)

INSTRUCTIONS	:	SEE NEXT PAGE
REQUIREMENTS	:	NONE

INSTRUCTIONS TO CANDIDATES:

- FORMULA SHEETS ATTACHED
- NO BOOKS, LECTURE NOTES, STUDY-, HOMEWORK- OR TUTORIAL MATERIAL ALLOWED
- UJ APPROVED CALCULATORS ALLOWED
- NO ANSWERS IN PENCIL OR RED INK WILL BE ACCEPTED
- ANSWER ALL 5 QUESTIONS IN ENGLISH
- SMOKING IS PROHIBITED DURING THE DURATION OF THE EXAM
- NOTE THE USE OF THE DECIMAL COMMA

QUESTION 1 (18 marks)

Consider an ideal shaft power gas turbine cycle with heat exchange and a **free power turbine**, schematically shown in figure 1. The cycle processes are as follows:

1-2: Isentropic compression in compressor (C)
2-5: Constant pressure heat exchange (HE)
5-3: Constant pressure heat input in combustion chamber (B)
3-4: Isentropic expansion in high pressure- (gas generator) turbine (T_{HP})
4-6: Constant pressure heat exchange (HE)
6-7: Isentropic expansion in low pressure- (free) turbine (T_{LP})
7-1: Constant pressure heat rejection

The following temperatures are equal: $T_4 = T_5$; $T_2 = T_6$;
The following pressures are equal: $P_1 = P_7$; $P_2 = P_3 = P_5$; $P_4 = P_6$

Derive an equation for the thermal efficiency of the cycle.

QUESTION 2 (13 marks)

The compressor outlet pressure of an ideal simple gas turbine is 850 kPa. The cycle work output is 557 132 J/kg and the heat rejected to atmosphere is 309 028 J/kg. The turbine entry temperature is 1 150 K. A cycle schematic is shown in figure 2.

Calculate the following:

(i) All the unknown states and the temperature- and pressure ratios of the cycle
(ii) Heat input and turbine, work input and work output per kg of air
(iii) Net work per kg of air
(iv) Thermal efficiency
(v) Maximum-to-minimum temperature ratio

Properties of air are given as follows: $C_P = 1\,005$ J/kgK; $\gamma = 1,4$

QUESTION 3 **(12 marks)**

(i) Consider the equation for thermal efficiency of the Dual cycle:

$$\eta_T = 1 - \frac{1}{r_V^{\gamma-1}}\left(\frac{\alpha c^{\gamma} - 1}{\gamma\alpha(c-1) + (\alpha - 1)}\right) \qquad (3.1)$$

Show that the equations for thermal efficiency of the Diesel- and Otto cycles are special cases of equation (3.1).

(ii) An engine operating on a Diesel cycle with an unknown compression ratio has a cutoff ratio of 2. Obtain an expression for the change in thermal efficiency if the cutoff ratio is increased by 50%, for a γ value of 1,4.

(iii) Calculate the percentage change in efficiency in (ii) above if the compression ratio is 17:1.

(iv) Calculate the cutoff ratio that gives minimum efficiency if the compression ratio is 17:1, as well as the value of the minimum efficiency, for a γ-value of 1,4.

QUESTION 4 **(30 marks)**

A four-stroke reciprocating internal combustion engine has a displacement volume of 1,8 *l* and a compression ratio of 11:1. At 1 500 m above sea level, it runs at 5 300 RPM at wide-open throttle, where its volumetric efficiency is 82%. The engine is "square", i.e. with equal bore and stroke. The connecting rod length is 250 mm.

Atmospheric pressure and temperature at sea level are 101 325 Pa and 288,15 K respectively. The temperature lapse rate is 0,0065 K/m above sea level. The gas constant and ratio of specific heat capacities of air are 287,1 J/kgK and 1,35 respectively.

Calculate

(i) Ambient temperature, pressure and density using the ISA atmosphere
(ii) Displacement volume per cylinder, bore and stroke
(iii) Crank radius and ratio of connecting rod length to crank radius
(iv) Piston displacement at 0°, 90° and 180° crank rotation after top dead centre
(v) Clearance volume per cylinder
(vi) Total volume per cylinder at 0°, 90° and 180° crank rotation after top dead centre
(vii) Air mass per cylinder at 0°, 90° and 180° crank rotation after top dead centre
(viii) Air mass flow through engine at 5 300 RPM and the ambient conditions determined in (i) above

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QUESTION 5 (34 marks)

The thrust F of a submerged boat propeller depends on the following parameters:

Water density ρ
Rotational speed N
Size D
Water viscosity μ
Axial velocity V of water through propeller

Form dimensionless groups with the above parameters of a boat propeller, using the *MLT* system of dimensions. Set up a table of dimensions of the parameters. Determine the number of dimensionless groups. Select primary parameters for the dimensional analysis. Form the dimensionless groups using Buckinghams π-theorem.

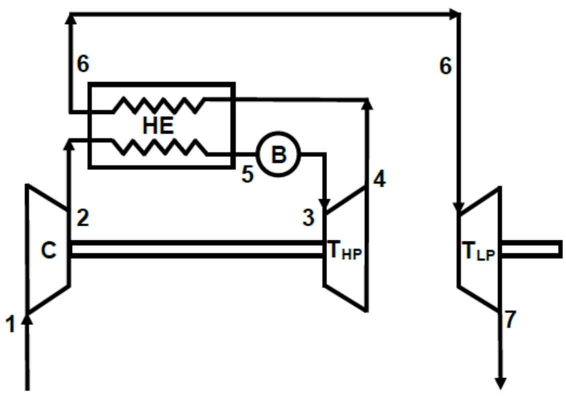

Figure 1: Ideal shaft power gas turbine cycle with heat exchange and a free turbine

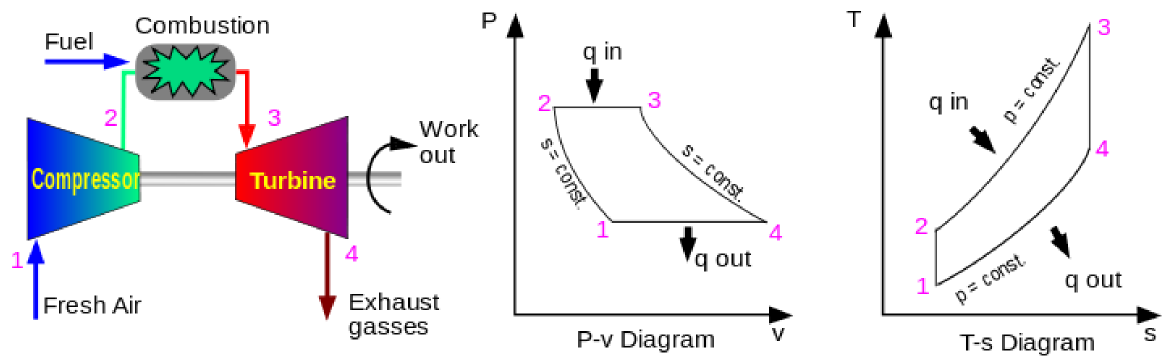

Figure 2: Ideal simple shaft power gas turbine cycle
https://en.wikipedia.org/wiki/Brayton_cycle

Annexure: Formula sheets

$$\eta = \frac{T'_{01} - T_a}{T_{01} - T_a}$$

$$c = \sqrt{\gamma RT}$$

$$\eta = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$M_a = c_a / c$$

$$\eta = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$P_{03} = P_{02}\left(1 - \frac{\Delta P_b}{P_{02}} - \frac{\Delta P_{ha}}{P_{02}}\right)$$

$$\eta = \frac{(FA)_{theor}}{(FA)_{actual}}$$

$$P_a = P_{04}\left(1 - \frac{\Delta P_{hg}}{P_{04}}\right)$$

$$\eta = \frac{T_{04} - T_5}{T_{04} - T'_5}$$

$$w = C_p(T_{01} - T_{02})$$

$$\eta = \frac{T_{04} - T_c}{T_{04} - T'_c}$$

$$w = C_p(T_{03} - T_{04})$$

$$\eta = \frac{P_{01} - P_a}{P_{0a} - P_a}$$

$$C_p - C_v = R$$

$$\eta = -\frac{w_C}{w_T} = -\frac{\dot{w}_C}{\dot{w}_T}$$

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

$$\eta = \frac{w_{net}}{q_{in}}$$

$$\frac{n-1}{n} = \frac{\gamma - 1}{\eta_p \gamma}$$

$$w_T = -\frac{w_C}{\eta} + \frac{w_{net}}{\eta}$$

$$\frac{n-1}{n} = \eta_p \frac{\gamma - 1}{\gamma}$$

$$\alpha = 1$$

$$\phi = (AF)_{stoich} / (AF)_{act}$$

$$\lambda = 1/\phi$$

$$SFC = 3600\dot{m}_f / \dot{W}$$

$$SFC = \dot{m}_f / \dot{W}$$

$$\eta = ISFC / BSFC$$

$$\eta = \frac{m_a}{\rho_a V_d}$$

$$\eta = \frac{60\dot{m}_a}{\alpha \rho_a V_d N} = \frac{60n\dot{m}_a}{\rho_a V_d N}$$

$$W_{61} = P_{in}(V_1 - V_6) = P_{in} V_d$$

$$W_{56} = P_{ex}(V_6 - V_5) = -P_{ex} V_d$$

$$(W_{pump})_{net} = (P_{in} - P_{ex})V_d$$

$$l_{max} < d_v / 4$$

$$C_{Dv} = A_{act} / A_{pass}$$

$$A_{pass} = \pi d_v l$$

$$A_i = \frac{\pi}{4} d_v^2 = CB^2 \frac{\bar{\dot{y}}_{max}}{c_i}$$

$$C \approx 1{,}3$$

$$P_1 v_1^{\gamma} = P_2 v_2^{\gamma}$$

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$\dot{Q} = \dot{m} C_v \Delta T$$

Formulae (continued)

$$\dot{Q} = \dot{m}(C_p + C_v)\Delta T$$

$$m_f q_{in} = (m_a + m_f)C_v \Delta T$$

$$dq = du + dw$$

$$dq = dh - v dP$$

$$dw = P dv$$

$$q_{in} + q_{out} = w_{net}$$

$$T(h) = T_{SL} - \lambda h$$

$$P(h) = P_{SL}\left(1 - \frac{\lambda}{T_{SL}} h\right)^{g/(\lambda R)}$$

$$\rho(h) = \rho_{SL}\left(1 - \frac{\lambda}{T_{SL}} h\right)^{\left(\frac{g}{\lambda R} - 1\right)}$$

$$P = \rho R T$$

$$C_p T_0 = C_p T + c^2/2$$

$$\frac{T_0}{T} = \left(\frac{P_0}{P}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(n-1)/n}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}}\right)^{(n-1)/n}$$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_{04}}{T_{03}} = \left(\frac{P_{04}}{P_{03}}\right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_c}{T_{04}} = \left(\frac{P_c}{P_{04}}\right)^{(\gamma-1)/\gamma}$$

$$T_c = \frac{2}{\gamma + 1} T_0$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$m_f q_{in} = (m_a + m_f)C_p \Delta T$$

$$(m_a + m_f)C_{Pg}(T_{03} - T_{02}) = \eta_b m_f Q_{LHV}$$

$$t = T_3/T_1$$

$$F = \dot{m}(c_j - c_a) + A_j(P_j - P_a)$$

$$F = \dot{m}(c_j - c_a)$$

$$SFC = \frac{3600(FA)}{F_s}$$

$C_p = 1\ 005$ J/kgK

$C_p = 1\ 148$ J/kgK

$R = 287{,}1$ J/kgK

$\gamma = 1{,}4$

$\gamma = 1{,}333$

$\lambda = -0{,}986$ °C/km

$\lambda = 0$ °C/km

$\lambda = 6{,}5$ °C/km

$g = 9{,}81$ m/s^2

$$\dot{m}_{fuel} = (FA)\dot{m}_a$$

$$\dot{m} = \frac{\dot{W}}{w}$$

$$\dot{m} = \rho c A$$

$$\dot{m} = F/F_s$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

Formulae (continued)

$$F_D = \dot{m}c_a = (\dot{m}_c + \dot{m}_h)c_a$$

$$F_N = F_G - F_D$$

$$w_{T\,LP} = -\frac{w_F}{\eta}$$

$$T_{05} - T_{06} = (1+B)\frac{C_{pa}}{\eta_m C_{pg}}(T_{02} - T_{01})$$

$$T_{SL} = 288{,}15\ K$$

$$P_{SL} = 101{,}325\ kPa$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \dot{m}_c / \dot{m}_h$$

$$\dot{m}_h = \dot{m}/(1+B)$$

$$\dot{m}_c = \dot{m}B/(1+B)$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$F_D = \dot{m}c_a = (\dot{m}_c + \dot{m}_h)c_a$$

$$\dot{W}_{T\,LP} = \dot{m}_h C_{pg}(T_{05} - T_{06})$$

$$\dot{W}_{T\,LP} = -\dot{W}_F / \eta_m$$

$$TP = \eta_{pr} SP + Fc_a$$

$$c = v_3 / v_x$$

$$EP = \frac{TP}{\eta_{pr}} = SP + \frac{Fc_a}{\eta_{pr}}$$

$$c = v_3 / v_2$$

$$EP_{take\,off} = \frac{TP}{\eta_{pr}} = SP + \frac{F}{8{,}5}$$

$$\alpha = P_x / P_2$$

$$\eta_{T\ Diesel} > \eta_{T\ Dual} > \eta_{T\ Otto}$$

$$s = a\cos\theta + \sqrt{r^2 - a^2\sin^2\theta}$$

$$y = (r+a) - (a\cos\theta + r\cos\phi)$$

$$R = r/a$$

$$\dot{y} = a\dot{\theta}\sin\theta\left(1 + \frac{\cos\theta}{\sqrt{R^2 - \sin^2\theta}}\right)$$

$$\dot{y} \approx a\Omega\left(\sin\theta + \frac{\sin 2\theta}{2R}\right)$$

$$\bar{\dot{y}} = 2S\,\frac{N}{60}$$

$$\frac{\dot{y}}{\bar{\dot{y}}} \approx \frac{\pi}{2}\left(\sin\theta + \frac{\sin 2\theta}{2R}\right)$$

$$V_d = V_{BDC} - V_{TDC}$$

$$V_d = \pi B^2 S/4$$

$$V_d = N_c \pi B^2 S/4$$

$$r_V = (V_c + V_d)/V_c$$

$$W = \int F\ dy$$

$$W = \int P\ dV$$

$$w = \int P\ dv$$

$$F = PA_p$$

$$A_p = \pi B^2/4$$

$$w_{net} = w_{gross} + w_{pump}$$

$$w_b = w_i - w_f$$

$$\eta_m = W_b / W_i$$

$$MEP = W/V_d$$

$$(IMEP)_{net} = (IMEP)_{gross} + PMEP$$

$$BMEP = (IMEP)_{net} - FMEP$$

$$BMEP = \eta_m (IMEP)_{net}$$

$$FMEP = (1 - \eta_m)(IMEP)_{net}$$

$$BMEP = 2\pi T_b / \alpha V_d$$

Formulae (continued)

$$\dot{W} = 2\pi NT/60$$

$$\dot{W}_{b\,engine} = (BMEP)V_d N\alpha/60$$

$$BMEP = 2\pi T_b/\alpha V_d$$

α = ½

$$m_f q_{in} = (m_a + m_f)C_v \Delta T$$

$$(AF)_{stoich} = 4{,}319\frac{32n+8m}{12n+1{,}008m}$$

$$\eta = 1 - \frac{1}{r_T}$$

$$\eta = 1 - \frac{1}{r_P^{(\gamma-1)/\gamma}}$$

$$\eta = 1 - \frac{r_T}{t}$$

$$\eta = \frac{2t - r_T + 1 - 2t/\sqrt{r_T}}{2t - r_T - t/\sqrt{r_T}}$$

$$w_{\text{net}}/C_P T_1 = 2t - r_T + 1 - 2t/\sqrt{r_T}$$

$$SFC = \frac{3600\,(FA)_{\text{actual}}}{w_{TP}}$$

$$\eta_T = \frac{w_{TP}}{Q_{HV}\,(FA)_{\text{actual}}}$$

$$\eta = 1 - \frac{1}{r_V^{\gamma-1}}$$

$$W_{\text{in}} = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$W_{\text{out}} = \frac{P_4V_4 - P_3V_3}{1-\gamma}$$

$$C_nH_m + \left(n + \frac{m}{4}\right)O_2 + 3{,}76\left(n + \frac{m}{4}\right)N_2 \rightarrow nCO_2 + \left(\frac{m}{2}\right)H_2O + 3{,}76\left(n + \frac{m}{4}\right)N_2$$