UNIVERSITY OF JOHANNESBURG

PROGRAM : BACCALAUREUS INGENERIAE
MECHANICAL ENGINEERING

SUBJECT : **STRENGTH OF MATERIALS 4A**

CODE : **SLR4A11**

DATE : JUNE 2019 (SUPPLEMENTARY EXAM)

DURATION : 3 HOURS

WEIGHT : 50 : 50

TOTAL MARKS : 100

EXAMINER : PROF RF LAUBSCHER (UJ)

MODERATOR : PROF C POLESE (WITS)

NUMBER OF PAGES : 8 PAGES

INSTRUCTIONS : QUESTION PAPERS MUST BE HANDED IN.

INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1

1.1 Define the deviatoric stress tensor. Obtain an expression for it in matrix form for a generalized stress state at a point. [5]

1.2 Find the shape functions Ni and Nj for a bar element with end nodes at i and j. Assume a linear interpolation function. Describe the difference between a simplex and a complex element? [5]

1.3 The basic finite element method may be applied to different physical problem areas. Illustrate this principal by deriving the system equation for a simple 1D thermal rod element if the following is given:

$$q = -KA\frac{dT}{dx}$$

[5]

1.4 Derive expressions for the principal stresses for the simple plane stress case if the basic transformation equations for plane stress are used as the starting point (see equation sheet). [10]

(25)

QUESTION 2

The following stress state exists at a point in the neck region of a socket adapter **(**E = 200 GPa, v = 0.3) as illustrated in Figure 1.

σ_{11} = 400 MPa, σ_{22} = 300 MPa, σ_{33} = 100 MPa, σ_{12} = 150 MPa and σ_{23} = 200 MPa.

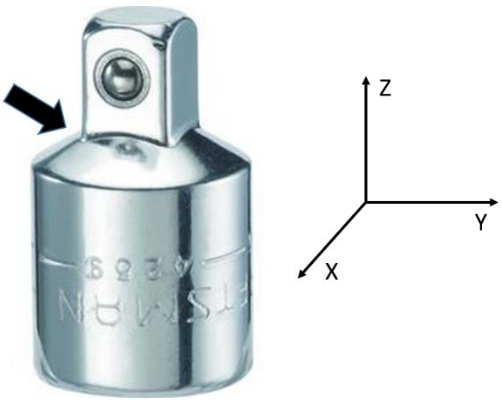

Figure 1 Socket adapter

2.1 Calculate the principal stresses. [6]

2.2 Calculate the maximum shear stress [4]

2.3 Determine the safety factor for the measured stress state (for both the von Mises and Tresca criteria) if the yield strength is 300 MPa. [6]

2.4 For a SN Curve as shown (Figure 2) estimate the fatigue life (assume that the current stress distribution is fully reversed for each load cycle). [5]

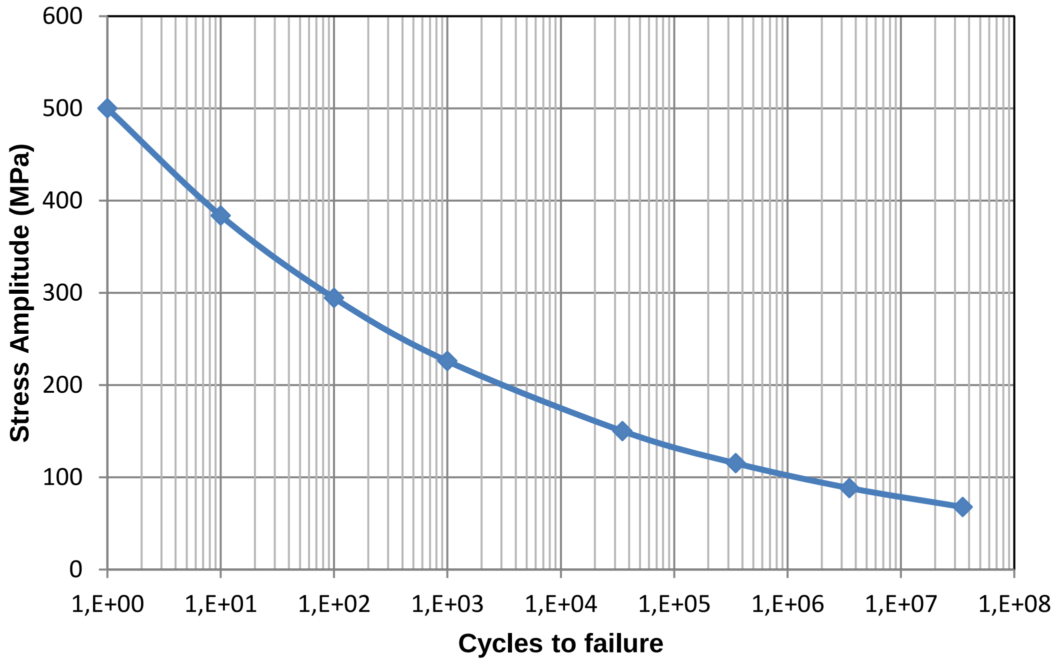

Figure 2 SN curve (Structural steel)

2.5 Calculate the 1st and 2nd invariant of the stress tensor? [4]

(25)

QUESTION 3

A dimensioned steel plate is presented in Figure 3. The plate has a thickness of 10 mm. The Modulus of Elasticity of the steel is 200 GPa (AISI 1020). A load of F = 10000 N is applied on the right hand face as shown (Figure 3). The plate is fixed at the left face. Use the finite element method to calculate the stresses across the plate and the maximum displacement of the free edge (hint: discretize the plate into appropriate simple one dimensional elements).

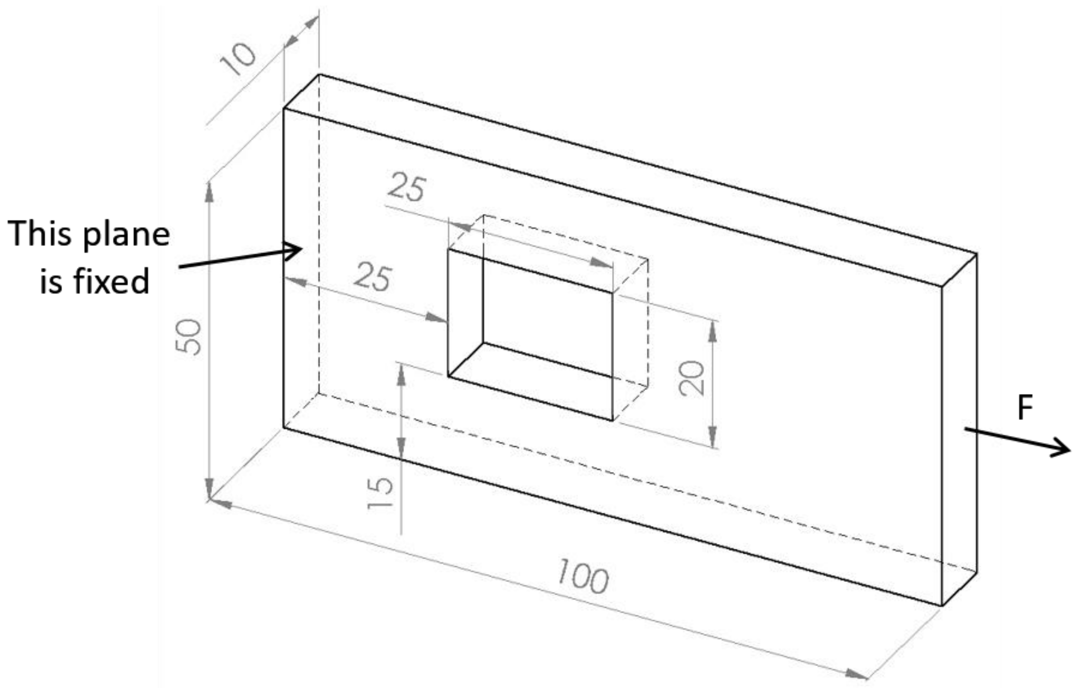

Figure 3 Fixed plate (dimensions are in mm). Load evenly distributed across the edge.

(20)

QUESTION 4

Consider the 2D plane strain element in Figure 4.

4.1 Calculate the shape functions associated with the three nodes i, j and k. [6]

4.2 Calculate the appropriate load vectors. [4]

4.3 Calculate the total force vector. [2]

4.4 Calculate the material property matrix. [2]

4.5 Calculate the matrix that relates the strains to the displacements. [2]

4.6 If the nodal displacements of the element are as follows:

$$\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 1 \\ 1 \end{Bmatrix} \times 10^{-3}\,\text{mm}$$

What are the displacements at the point (2,2)? [4]

4.7 Calculate the stress state of the element. [5]

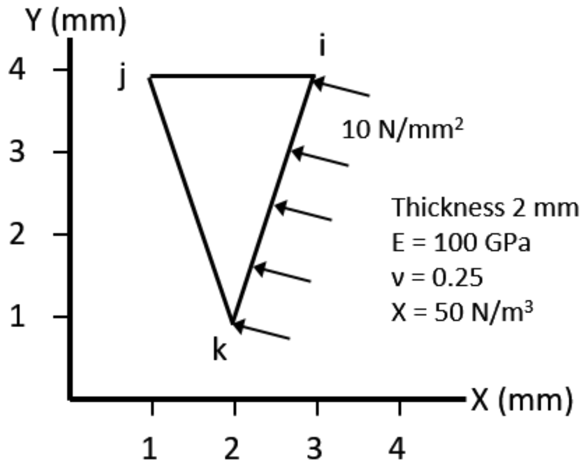

Figure 4 Plane strain triangular element

(25)

Equation sheet

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \qquad \lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \qquad \frac{E}{1+\upsilon} = 2G$$

$$\varepsilon'_{kl} = a_{ki}a_{lj}\varepsilon_{ij} \qquad \sigma'_{kl} = a_{ki}a_{lj}\sigma_{ij}$$

$$\cos^2\theta = \frac{(\cos 2\theta + 1)}{2} \qquad \sin^2\theta = \frac{(1-\cos 2\theta)}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sigma^3 - (\sigma_{11} + \sigma_{22} + \sigma_{33})\,\sigma^2 + \left(\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2\right)\sigma$$
$$-\left(\sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2\right) = 0$$

$$\varepsilon^3 - (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\,\varepsilon^2 + \left(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{33} - \varepsilon_{12}^2 - \varepsilon_{23}^2 - \varepsilon_{13}^2\right)\varepsilon$$
$$-\left(\varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{31} - \varepsilon_{11}\varepsilon_{23}^2 - \varepsilon_{22}\varepsilon_{13}^2 - \varepsilon_{33}\varepsilon_{12}^2\right) = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

$$t^3 - pt + q = 0$$

$$x = t - \frac{b}{3a} \quad and \quad p = \frac{3ac - b^2}{3a^2} \quad and \quad q = \frac{2b^3 - 9abc + 27a^2d}{27a^3}$$

$$t_k = 2\sqrt{-\frac{p}{3}}\cos\left(\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - k\frac{2\pi}{3}\right) \quad \text{for} \quad k = 0,1,2.$$

$$\sigma_0 = \sigma_1 - \sigma_3$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2\right)\right]^{1/2}$$

$$J_2 = \frac{1}{6}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2\right)\right]$$

$$[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad [k] = K\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad [k] = \frac{AK}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\sigma'_{11} = \frac{\sigma_{11} + \sigma_{22}}{2} + \frac{\sigma_{11} - \sigma_{22}}{2}\cos 2\theta + \sigma_{12}\sin 2\theta$$

$$\sigma'_{22} = \frac{\sigma_{11} + \sigma_{22}}{2} - \frac{\sigma_{11} - \sigma_{22}}{2}\cos 2\theta - \sigma_{12}\sin 2\theta$$

$$\sigma'_{12} = \frac{\sigma_{22} - \sigma_{11}}{2}\sin 2\theta + \sigma_{12}\cos 2\theta$$

$$\varepsilon'_{11} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2}\cos 2\theta + \varepsilon_{12}\sin 2\theta$$

$$\varepsilon'_{22} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} - \frac{\varepsilon_{11} - \varepsilon_{22}}{2}\cos 2\theta - \varepsilon_{12}\sin 2\theta$$

$$\varepsilon'_{12} = \frac{\varepsilon_{22} - \varepsilon_{11}}{2}\sin 2\theta + \varepsilon_{12}\cos 2\theta$$

$$\left[[B]^T[D][B]tA\right]\begin{Bmatrix} u \\ v \end{Bmatrix} = \frac{\alpha Et(\Delta T)}{2(1-v)}\begin{Bmatrix} b_i \\ c_i \\ b_j \\ c_j \\ b_k \\ c_k \end{Bmatrix} + \frac{At}{3}\begin{Bmatrix} X \\ Y \\ X \\ Y \\ X \\ Y \end{Bmatrix} + \frac{t}{2}\left\{ H_{ij}\begin{Bmatrix} p_x \\ p_y \\ p_x \\ p_y \\ 0 \\ 0 \end{Bmatrix} + H_{jk}\begin{Bmatrix} 0 \\ 0 \\ p_x \\ p_y \\ p_x \\ p_y \end{Bmatrix} + H_{ki}\begin{Bmatrix} p_x \\ p_y \\ 0 \\ 0 \\ p_x \\ p_y \end{Bmatrix} \right\} + \{P\}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\varepsilon_z = -\frac{\upsilon}{E}\left(\sigma_x + \sigma_y\right) + \alpha\Delta T$$

$$\varepsilon_0 = \alpha\Delta T\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right) - E\alpha\Delta T$$

$$\varepsilon_0 = (1+\nu)\alpha\Delta T\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{\varepsilon\} = [B]\{U\}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2A}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix}\begin{Bmatrix} u_{2i-1} \\ u_{2i} \\ u_{2j-1} \\ u_{2j} \\ u_{2k-1} \\ u_{2k} \end{Bmatrix} = [B]\{U\}$$

$$A = \frac{1}{2}\left(x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j\right) = \frac{1}{2}\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$

$$a_i = x_j y_k - x_k y_j \qquad b_i = y_j - y_k \qquad c_i = x_k - x_j$$

$$a_j = x_k y_i - x_i y_k \qquad b_j = y_k - y_i \qquad c_j = x_i - x_k$$

$$a_k = x_i y_j - x_j y_i \qquad b_k = y_i - y_j \qquad c_k = x_j - x_i$$

$$\phi = N_i\phi_i + N_j\phi_j + N_k\phi_k = [N]\{\Phi\}$$

$$N_i = \left(a_i + b_i x + c_i y\right)/\,2A$$
$$N_j = \left(a_j + b_j x + c_j y\right)/\,2A$$
$$N_k = \left(a_k + b_k x + c_k y\right)/\,2A$$

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = [N]\{U\}$$