UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE *MECHANICAL ENGINEERING*
SUBJECT	:	**STRENGTH OF MATERIALS 4A**
CODE	:	**SLR4A11**
DATE	:	JUNE 2019 (EXAM)
DURATION	:	3 HOURS
WEIGHT	:	50 : 50
TOTAL MARKS	:	100

EXAMINER	:	PROF RF LAUBSCHER (UJ)
MODERATOR	:	PROF C POLESE (WITS)
NUMBER OF PAGES	:	9 PAGES

INSTRUCTIONS	:	QUESTION PAPERS MUST BE HANDED IN.

INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

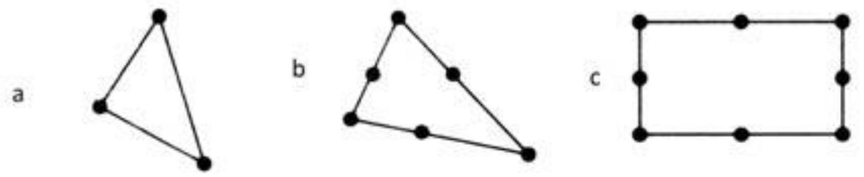
QUESTION 1

1.1 What are the basic kinematic and material assumptions usually made when conducting a structural evaluation of parts or structures? [4]

1.2 What are the basic force types that may act on a body? Give 2 examples of each. [6]

1.3 An infinitesimal cube has six sides. Each side is associated with one normal stress and two shear stress components. This implies a total of 18 components of stress. Briefly show how this may be reduced to only 6. [9]

1.4 Write down interpolation functions for the following elements: [6]



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QUESTION 2

A new shear pin load cell is to be designed. It uses appropriately located strain gauges to measure strain and therefore load. The best sensitivity will be obtained where the largest strain is measured for a given load. As an initial step a strain gauge rosette (3 strain gauges) is placed in the general vicinity of where the maximum strain is to be expected. Essentially we want to know how should the strain gauges used for the production model be alighted for maximum sensitivity.

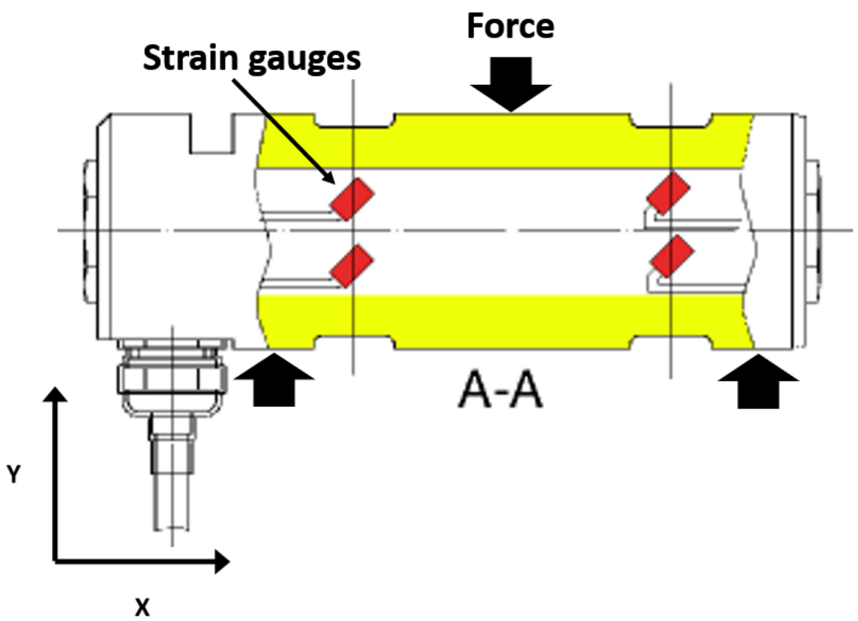

Figure 1 Schematic of shear pin load cell.

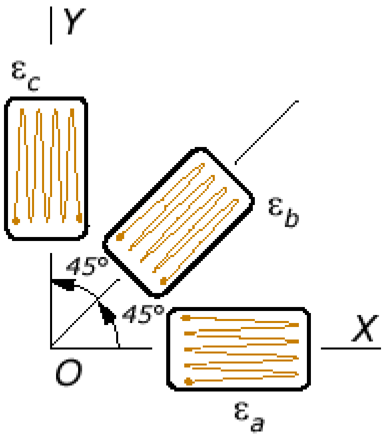

Figure 2 Strain gauge rosette.

The following strain readings are obtained (E = 200 GPa, v = 0.3):

- $\varepsilon_a = 300\times10^{-6}$
- $\varepsilon_b = 200\times10^{-6}$
- $\varepsilon_c = 400\times10^{-6}$

2.1 Transform the strains obtained by the strain gauges to a general strain state relative to the coordinate system as shown in Figure 2. [5]

2.2 How should the strain gauges be aligned (rotated) to obtain the maximum sensitivity (linear strain)? What is the maximum expected linear strain? [5]

2.3 Derive the constitutive equations for the plane stress case by expanding from Hooke's law in tensor notation. [5]

2.4 If the general strain state as obtained in Q2.1 is $\varepsilon_{11} = 100\times10^{-6}$, $\varepsilon_{22} = 100\times10^{-6}$ and $\varepsilon_{12} = 100\times10^{-6}$. Calculate the general stress state (relative to axis X-Y). [3]

2.5 If the general stress state as calculated in Q2.4 is σ_{11} = 200 MPa, σ_{22} = 400 MPa and σ_{12} = 200 MPa, what is the expected maximum shear stress. [7]

2.6 If the material yield stress is σ_0 = 300 MPa, check for yielding using both the von Mises and Tresca Criteria (assume stress state as presented in Q2.5). [5]

(30)

QUESTION 3

A solid one dimensional bar system is shown in Figure 3. The base (L_1) is Nylon whereas the extension (L_2) is PVC. Use the finite element method to calculate the 1) end displacement, 2) the reaction force on the wall and the 3) average stresses.

L_1 = 50 mm, L_2 = 60 mm, F_1 = 2000 N, F_2 = 4000 N, A_1 = 100 mm^2, A_2 = 75 mm^2 and E_{Nylon} = 2 GPa, E_{PVC} = 3 GPa.

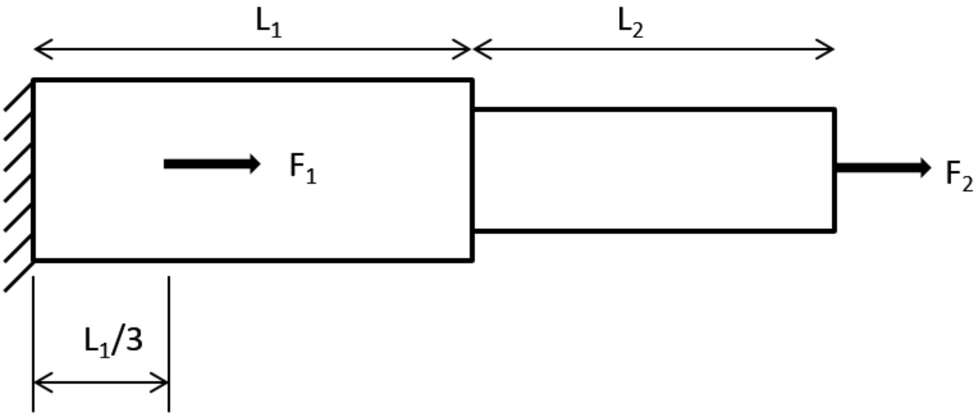

Figure 3 1D Bar system

(20)

QUESTION 4

Consider the 2D plane stress element in Figure 4.

4.1 Calculate the shape functions associated with the three nodes i, j and k. [6]

4.2 Calculate the appropriate load vectors. [4]

4.3 Calculate the total force vector. [2]

4.4 Calculate the material property matrix. [2]

4.5 Calculate the matrix that relates the strains to the displacements. [2]

4.6 If the nodal displacements of the element are as follows:

$$\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{Bmatrix} \times 10^{-3}\,\text{mm}$$

What are the displacements at the point (2,2)? [4]

4.7 Calculate the stress state of the element. [5]

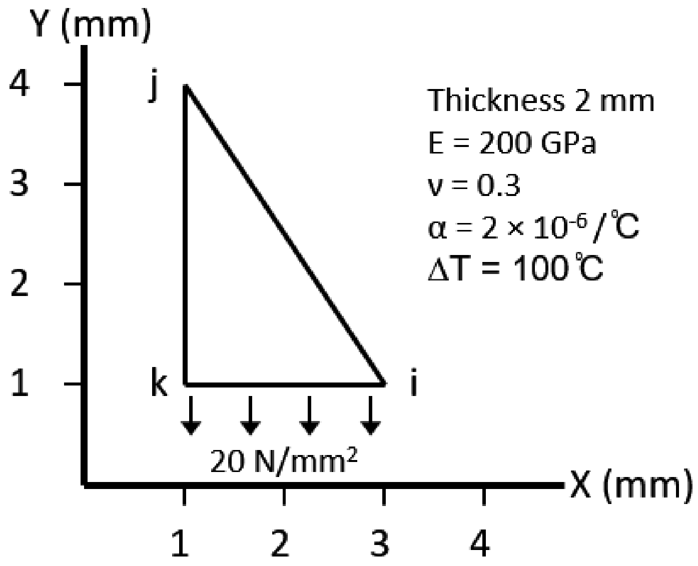

Figure 4 Plane stress triangular element

(25)

Equation sheet

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \qquad \lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \qquad \frac{E}{1+\upsilon} = 2G$$

$$\varepsilon'_{kl} = a_{ki}a_{lj}\varepsilon_{ij} \qquad \sigma'_{kl} = a_{ki}a_{lj}\sigma_{ij}$$

$$\cos^2\theta = \frac{(\cos 2\theta + 1)}{2} \qquad \sin^2\theta = \frac{(1-\cos 2\theta)}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sigma^3 - (\sigma_{11} + \sigma_{22} + \sigma_{33})\,\sigma^2 + \left(\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2\right)\sigma$$
$$-\left(\sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2\right) = 0$$

$$\varepsilon^3 - (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\,\varepsilon^2 + \left(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{33} - \varepsilon_{12}^2 - \varepsilon_{23}^2 - \varepsilon_{13}^2\right)\varepsilon$$
$$-\left(\varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{31} - \varepsilon_{11}\varepsilon_{23}^2 - \varepsilon_{22}\varepsilon_{13}^2 - \varepsilon_{33}\varepsilon_{12}^2\right) = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

$$t^3 - pt + q = 0$$

$$x = t - \frac{b}{3a} \quad and \quad p = \frac{3ac - b^2}{3a^2} \quad and \quad q = \frac{2b^3 - 9abc + 27a^2 d}{27a^3}$$

$$t_k = 2\sqrt{-\frac{p}{3}}\cos\left(\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - k\frac{2\pi}{3}\right) \quad \text{for} \quad k = 0,1,2.$$

$$\sigma_0 = \sigma_1 - \sigma_3$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2\right)\right]^{1/2}$$

$$J_2 = \frac{1}{6}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2\right)\right]$$

$$[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad [k] = K\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad [k] = \frac{AK}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\sigma'_{11} = \frac{\sigma_{11}+\sigma_{22}}{2} + \frac{\sigma_{11}-\sigma_{22}}{2}\cos 2\theta + \sigma_{12}\sin 2\theta$$

$$\sigma'_{22} = \frac{\sigma_{11}+\sigma_{22}}{2} - \frac{\sigma_{11}-\sigma_{22}}{2}\cos 2\theta - \sigma_{12}\sin 2\theta$$

$$\sigma'_{12} = \frac{\sigma_{22}-\sigma_{11}}{2}\sin 2\theta + \sigma_{12}\cos 2\theta$$

$$\varepsilon'_{11} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta + \varepsilon_{12}\sin 2\theta$$

$$\varepsilon'_{22} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} - \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta - \varepsilon_{12}\sin 2\theta$$

$$\varepsilon'_{12} = \frac{\varepsilon_{22}-\varepsilon_{11}}{2}\sin 2\theta + \varepsilon_{12}\cos 2\theta$$

$$\left[[B]^T[D][B]tA\right]\begin{Bmatrix}u\\v\end{Bmatrix} = \frac{\alpha Et(\Delta T)}{2(1-v)}\begin{Bmatrix}b_i\\c_i\\b_j\\c_j\\b_k\\c_k\end{Bmatrix} + \frac{At}{3}\begin{Bmatrix}X\\Y\\X\\Y\\X\\Y\end{Bmatrix} + \frac{t}{2}\left\{H_{ij}\begin{Bmatrix}p_x\\p_y\\p_x\\p_y\\0\\0\end{Bmatrix} + H_{jk}\begin{Bmatrix}0\\0\\p_x\\p_y\\p_x\\p_y\end{Bmatrix} + H_{ki}\begin{Bmatrix}p_x\\p_y\\0\\0\\p_x\\p_y\end{Bmatrix}\right\} + \{P\}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\varepsilon_z = -\frac{\upsilon}{E}(\sigma_x+\sigma_y) + \alpha\Delta T$$

$$\varepsilon_0 = \alpha\Delta T\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$

$$\sigma_z = \nu(\sigma_x+\sigma_y) - E\alpha\Delta T$$

$$\varepsilon_0 = (1+\nu)\alpha\Delta T\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$

$$\{\varepsilon\} = [B]\{U\}$$

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \frac{1}{2A}\begin{bmatrix}b_i & 0 & b_j & 0 & b_k & 0\\ 0 & c_i & 0 & c_j & 0 & c_k\\ c_i & b_i & c_j & b_j & c_k & b_k\end{bmatrix}\begin{Bmatrix}u_{2i-1}\\u_{2i}\\u_{2j-1}\\u_{2j}\\u_{2k-1}\\u_{2k}\end{Bmatrix} = [B]\{U\}$$

$$A = \frac{1}{2}\left(x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j\right) = \frac{1}{2}\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$

$$a_i = x_j y_k - x_k y_j \qquad b_i = y_j - y_k \qquad c_i = x_k - x_j$$

$$a_j = x_k y_i - x_i y_k \qquad b_j = y_k - y_i \qquad c_j = x_i - x_k$$

$$a_k = x_i y_j - x_j y_i \qquad b_k = y_i - y_j \qquad c_k = x_j - x_i$$

$$\phi = N_i \phi_i + N_j \phi_j + N_k \phi_k = [N]\{\Phi\}$$

$$N_i = \left(a_i + b_i x + c_i y\right) / 2A$$
$$N_j = \left(a_j + b_j x + c_j y\right) / 2A$$
$$N_k = \left(a_k + b_k x + c_k y\right) / 2A$$

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = [N]\{U\}$$