SURNAME:________________________ INITIALS:________________________

STUDENT NUMBER:____

UNIVERSITY of JOHANNESBURG

Physics 2Y (PHY00Y2/PHY002Y) January Supplementary Exam

Thermal Physics

	Student's mark	Questions' mark
Q1		**22**
Q2		**17**
Q3		**12**
Total		**51**

Date: January 2020

Examiner: Mr L. Nyadzani

Moderator: Prof. E. Carleschi

Time: 105 Minutes

Pencils and cell-phones are not allowed.
Answer all the questions.
This paper consists of 9 pages including the information sheet.

Leave any calculations in numbers if you don't have a calculator.

Where necessary, explain what you are doing in derivations.

IF YOU DO NOT UNDERSTAND ANY OF THE LANGUAGE USED, ASK!!!

IF YOU NEED MORE SPACE WRITE ON THE BACK OF THE PAGE

Do not write in the margins

Question 1 [22]

1.1 For an ideal gas of one molecule in a smooth cylinder of volume V, with a piston at one end, show that the average pressure is given by: $\overline{P} = \dfrac{mv_x^2}{V}$. [6]

1.2 How are the heat capacity at constant pressure and the heat capacity at constant volume related, for an ideal gas? [3]

1.3 State the equipartition theorem. [2]

1.4 Prove the following equation for adiabatic compression. You may assume the integral formula for quasistatic work.

$VT^{f/2} = \text{constant}$

where f are the number of degrees of freedom per molecule. [7]

1.5 Derive an approximate formula for the mean free path of a molecule in a dilute ideal gas. Explain all steps. [4]

Question 2 [17]

2.1 Explain the following terms, giving a suitable example for each:

(i) microstate [1]

(ii) macrostate [1]

(iii) multiplicity [1]

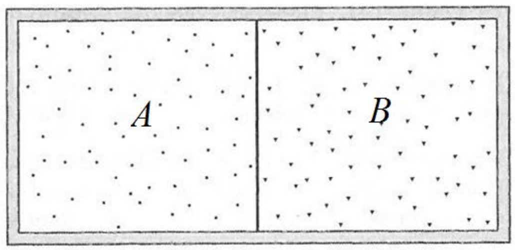
2.2 Derive an expression for the multiplicity of a two-state system, such as a set of coins. Explanations count marks. [6]

2.3 Two Einstein solids share 40 units of energy. Solid A has 15 oscillators and solid B has 20 oscillators.

(i) How many possible macrostates are there? [1]

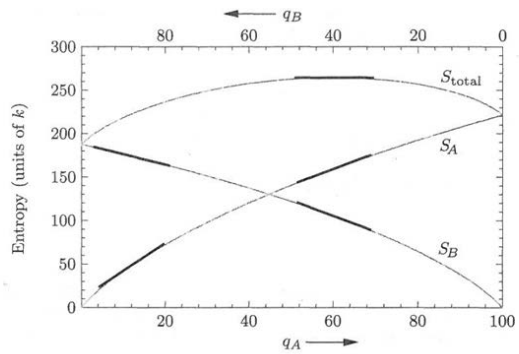
(ii) What is the probability of finding all the energy in A? [3]

2.4 Suppose that we have two different monoatomic ideal gases, A and B, each with the-same energy, volume and number of particles. They occupy the two halves of a chamber, separated by a partition, as shown in the below figure. Calculate the entropy increase if the partition is removed. [4]

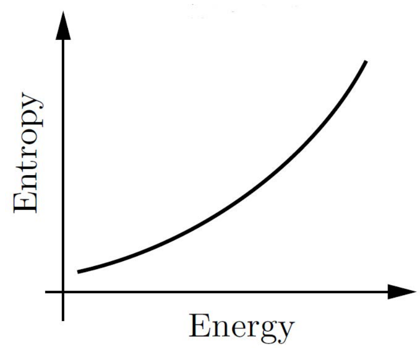


Question 3 [12]

3.1 Using the figure below (describing two weakly coupled Einstein oscillators A and B), give three arguments that lead to the definition of temperature as $T \equiv \left(\dfrac{\partial S}{\partial U}\right)^{-1}$. You must write down any relevant formulas. [6]



3.2 Can a system with a concave-up entropy-energy graph, ever be in stable thermal equilibrium with another system? Explain. The graph is shown in the
gure below. [2]



3.3 Use the expression for the entropy of a monoatomic ideal gas (see information sheet), to calculate the energy of this gas. Explain why the result is what you expect. [4]

END of QUESTIONS

INFORMATION SHEET

$$R = 8.31\frac{\text{J}}{\text{mol.K}} \quad ; \quad N_A = 6.022\times10^{23}\ \text{mol}^{-1} \quad ; \quad \text{Atmospheric pressure} = 1.03{\times}10^5\ \text{Pa}$$

$$\text{Boltzmann's constant: } k = \frac{R}{N_A} = 1.381\times10^{-23}\text{J/K}$$

$$\text{Equipartition theorem: } U_{\text{per molecule}} = \frac{f}{2}kT$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$\text{Adiabatic compression: } VT^{f/2} = \text{constant} \text{ and } V^\gamma P = \text{constant where } \gamma = (f+2)/f$$

$$\text{Fourier heat conduction law: } \frac{Q}{\Delta t} = -k_t A\frac{dT}{dx}$$

$$\text{Two-state system multiplicity: } \Omega(N,n) = \frac{N!}{n!\cdot(N-n)!} = \binom{N}{n}$$

$$\text{Multiplicity of an Einstein solid: } \Omega(N,q) = \frac{(q+N-1)!}{q!\cdot(N-1)!} = \binom{q+N-1}{q}$$

$$\text{Stirling's approximation: } N! \approx N^N e^{-N}\sqrt{2\pi N} \text{ and } \ln N! \approx N\ln N - N$$

$$\text{Approximate form of the Heisenberg uncertainty principle: } (\Delta x)(\Delta p_x) \gtrapprox h$$

$$\text{Sackur-Tetrode equation: } S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$c_V(\text{water}) = 4186 \text{ J/kg.K}$$

$$\frac{1}{T} \equiv \left(\frac{\partial S}{\partial U}\right)_{N,V}$$

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}) \quad ; \quad \cosh x = \tfrac{1}{2}(e^x + e^{-x}) \quad ; \quad \tanh x = (\sinh x)/(\cosh x)$$