FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING: MINERAL SURVEYING/EXTRACTION METALLURGY

MODULE MNM31-1
NUMERICAL METHODS

CAMPUS DFC

MAIN EXAMINATION

DATE: 09 /11/2019 **SESSION: 8:H30 - 11:30**

ASSESSOR **DR B.P. NTSIME**

INTERNAL MODERATOR **DR D.M. MOTHIBI**

DURATION: 3 HRS **MARKS: 75 MARKS**

SURNAME & INITIALS:

STUDENT NUMBER:

COURSE:

CONTACT NO:

INSTRUCTIONS : ANSWER ALL QUESTIONS BY CREATING APPROPRIATE MATHEMATICA CODES
NO EXTERNAL STORAGE DEVICES ARE PERMITTED
NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ALLOWED

REQUIREMENTS: NONE

Question 1

a) Show graphically that $f(x) = x\log_{10} x - 1.2$, has the root in [1,2]. [5]

b) Use the following methods to find the root of f within a tolerance criterion $|f(x)| < 10^{-6}$, determining the number of iterations required

(i) the secant method [10]

(iii) the Newton-Raphson method with x_0=1.06 [10]

Question 2

a) (i) Use the inbuilt *Mathematica* solver to solve the following system of equations.

$$x_1 + 2x_2 + 2x_3 = 4$$

$$4x_3 - x_2 + 3x_1 = 25 \qquad [5]$$

$$3x_1 - x_3 + 2x_2 = -6$$

b) Consider the data presented in the table below

x_i	f_i
3	0
6	0.0872
9	0.1736
12	0.2588
15	0.3420
18	0.4426

(i) Find the polynomial of highest possible degree that interpolates f.

(ii) Find the polynomial of degree 2, $P_2(x)$, that best fits the data in the least squares sense.

(iii) Graph the interpolating polynomial, P_2 and the data points on the same axes.

[10]

Question 3

a) Solve the following system on linear equations using the Gauss Seidel method. Terminate iterations when the infinity norm of the residual is 10^{-6}. Use the ZERO vector as starting value.

$$2p - 3q = 4 - 6t$$

$$4p + q = -3 = 3q + t - 2 \qquad [10]$$

Question 4

Solve the set of non-linear equations

$6x^4 + y^4 = 6x^4 y^4$ and, $12x^4 + 2y^4 = 12x^4 y^4$

using Newton's method with starting values for $x_0 = 0.5$ and $y_0 = 0.5$. Terminate the method when $\| f(x) \|_\infty < 10^{-4}$. [10]

Question 5

5.1 Use trapezoidal rule to approximate

$$\int_0^{0.5} \frac{1}{\sqrt{1-x}}\, dx$$

using 50 sub-intervals [5]

5.2 Use Euler's method with a step size of $h = 0.4$ to find an approximate solution of the following IVP

$$xy' - 3y = 0\ , \qquad y(-3) = 2$$

over $-3 \le x \le 5$. Graph the solution, as well as the analytic solution, which is $y(x) = \frac{-2x^3}{27}$.

[10]