| SM | |
|----|--|
| EM | |
| FM | |

**FACULTY OF SCIENCE**

| | |
|--|--|
| **DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS** <br> **B ENG TECH IN EXTRACTION METALLURGY AND PHYSICAL METALLURGY** <br><br> **MODULE:**      **MATHED1** <br> **COURSE:**      **MATHEMATICS X1** <br> **CAMPUS:**      **DFC** <br> **EXAM:**        **MAIN EXAMINATION NOVEMBER 2019** | |

**DATE:**         **TUESDAY 19 NOVEMBER 2019**

**TIME:**         **08:30 – 11:30**

**EXAMINERS:**                              **DR R. DURANDT & MS T. KAISARA**

**INTERNAL MODERATOR:**              **DR S. SIMELANE**

**DURATION:**    **3 HOURS**                        **MARKS:   100**

| | |
|--|--|
| **STUDENT NUMBER** | |
| **SURNAME** | |
| **INITIALS** | |
| **CONTACT NUMBER** | |

| **LECTURER** | **GROUP** | **SELECT THE APPLICABLE OPTION – X** |
|--------------|-----------|--------------------------------------|
| **DR R. DURANDT** | **EXTRACTION METALLURGY** | |
| **MS T. KAISARA** | **PHYSICAL METALLURGY** | |

**NUMBER OF PAGES:**     **17 PAGES INCLUDING FRONT PAGE.**

**INSTRUCTIONS:**

- **QUESTIONS ARE ARRANGED IN 2 DISTINCT SECTIONS (A & B)**
- **ANSWER ALL THE QUESTIONS IN EITHER BLACK OR BLUE PEN**
- **NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ARE ALLOWED**
- **NO FORMULAE BOOKLETS WILL BE AVAILABLE**

Mathematics X1: November 2019 Examination

**SECTION A: PRECALCULUS**                                                   **[Total 50]**
**SECTION A1: SHORT ANSWER QUESTIONS**                      **[Total 24]**
**IN THIS SECTION, GIVE YOUR FINAL ANSWER ROUNDED TO TWO DECIMAL PLACES WHERE APPLICABLE. YOU CAN SHOW SOME CALCULATIONS, ALTHOUGH THIS IS NOT ESSENTIAL. USE THE BLANK PAGES ON THE OPPOSITE SIDE FOR ROUGH WORK.**

_____

**Question 1**                                             **(1 mark each = 4 marks)**
**USE THE TABLE BELOW TO <u>MARK THE LETTER</u> (X) CORRESPONDING TO THE CORRECT ANSWER FOR EACH QUESTION.**

| 1.1 | A | B | C | D |
|-----|---|---|---|---|
| 1.2 | A | B | C | D |
| 1.3 | A | B | C | D |
| 1.4 | A | B | C | D |

1.1     Evaluate $\left[\sqrt{3}(cos10° + jsin10°)\right]^9$

    A. $3^{\frac{9}{2}}(1 + j)$

    B. $3^{\frac{9}{2}}$

    C. $j3^{\frac{9}{2}}$

    D. $3^{\frac{2}{9}}j$

1.2     Solve $y = \log_4 28 + \log_3 24$

    A. 0.35

    B. 1.08

    C. 5.30

    D. 0.44

1.3     Given $f(x) = -5x$ and $g(x) = 2x^2$ , find the composite function $(f \circ g)(x)$

    A. $25x^2$

    B. $-10x^2$

    C. $50x^2$

    D. $-10x^3$

1.4     Find the inverse of the function $x = t^3$

    A. $x = 3t$

    B. $x = \frac{t}{3}$

    C. $x = t^3$

    D. $x = \sqrt[3]{t}$

**Question 2**
Indicate **True** or **False** and, if False, give a reason or explanation for your answer:     (2)
_The equation_ $\left(x - \frac{1}{8}\right)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{25}{81}$ _represents a circle of radius_ $\frac{5}{9}$ _with centre_ $\left(\frac{1}{8}, \frac{2}{3}\right)$

**Question 3**

A man, who is 2m tall, stands on horizontal ground 30m from a tree. The angle of elevation of the top of the tree from his head is 28°.

3.1     Sketch the diagram to represent the situation.                                    (1)

3.2     Estimate the height of the tree.                                                    (2)

**Question 4**

Find the domain of the function:                                                           (2)

$$P(x) = \frac{1}{\sqrt{9 - x^2}}$$

**Question 5**

An instantaneous value of voltage in an a.c. circuit at any time t seconds is given by:

$$V = 340sin(50\pi t - 0.541)$$

5.1     Determine the:                                                                     (3)
        a)      amplitude

        b)      periodic time (period)

        c)      phase angle

5.2     Find the value of voltage when $t = 0$.                                          (1)

5.3 Sketch one cycle of the wave-form and show readings on the graph for all key points.

(1)

## Question 6
Given $E = \frac{1}{2}mv^2 + mgh$ , make $m$ the subject of the formula. (2)

## Question 7
A point P has polar coordinates $\left(3, \frac{\pi}{4}\right)$. What are the rectangular coordinates for the point P? (2)

## Question 8
If $z_1 = 2\sqrt{3}\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$ and $z_2 = 3\sqrt{3}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$, find:

8.1 $\dfrac{z_2}{z_1}$ . (2)

8.2 $(z_1)^3$ (2)

**SECTION A2: APPLICATION QUESTIONS**          **[Total 26]**
**IN THIS SECTION, SHOW <u>ALL THE STEPS</u> TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.**
**USE THE BLANK PAGES (ON THE OPPOSITE SIDE) FOR ROUGH WORK.**
_____

**Question 9**
Find the equation of an ellipse that has foci $(\pm 5, 0)$ and the length of the major axis is 12 units.

(3)

**Question 10**
Given the Binomial series,

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + \cdots$$

Expand and find the first four terms of $\dfrac{1}{(2+t)^3}$ .

(5)

**Question 11**

Prove the identity: (5)

$$\frac{cot\theta + tan\theta}{sec\theta} = cosec\theta$$

**Question 12**

Solve for $x$: (5)
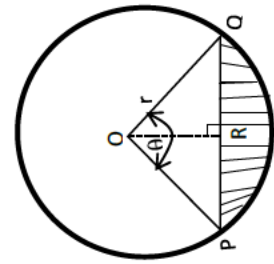
$$(15.7)^{2x-1} = (7.4)^{x+2}$$

**Question 13**

Use Cramer's Rule to solve for $x$ in the system: (5)

$$\begin{cases} 2x - 3y + 4z &= 1 \\ x + 6z &= 0 \\ 3x - 2y &= 5 \end{cases}$$

## Question 14

A segment of a circle (centre point at O) has base (chord PQ) of length 225.74 mm. The triangle POQ has height (OR) of 175 mm. Calculate:



14.1    The radius of the circle. (1)

14.2    The angle $\theta$ in radians. (1)

14.3    The area of the shaded segment. (1)

**SECTION B: CALCULUS** [Total 50]
**SECTION B1: SHORT ANSWER QUESTIONS** [Total 21]
**IN THIS SECTION, GIVE YOUR FINAL ANSWER ROUNDED TO TWO DECIMAL PLACES WHERE APPLICABLE. YOU CAN SHOW SOME CALCULATIONS, ALTHOUGH THIS IS NOT ESSENTIAL. USE THE BLANK PAGES ON THE OPPOSITE SIDE FOR ROUGH WORK.**

_____

**Question 1**
Differentiate the function with respect to $x$: (2)

$$y = 3e^x + \frac{4}{\sqrt[3]{x}}$$

**Question 2**
Differentiate the function with respect to $x$: (2)

$$y = (sinx)lnx$$

**Question 3**
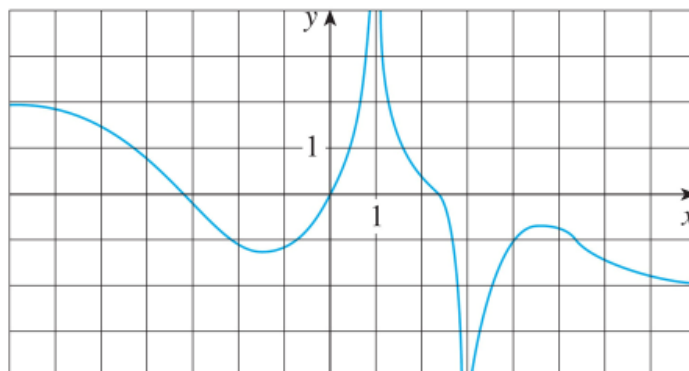Differentiate the function with respect to $x$: (2)

$$y = tan(3^x)$$

**Question 4**
Differentiate the function with respect to $x$: (1)

$$y = cos(\pi)$$

**Question 5**

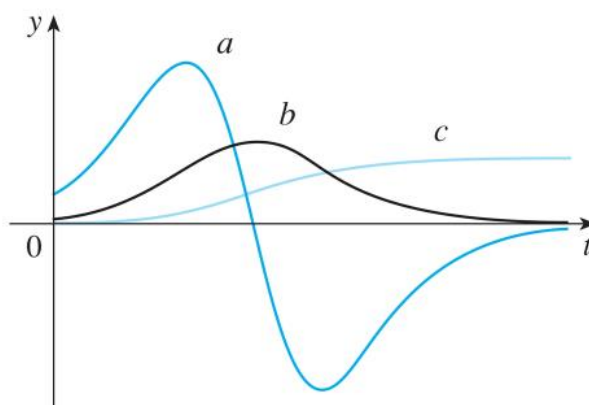Investigate the given graph of a function $f$ and then answer the questions below:     (4)



a) $\lim\limits_{x \to -\infty} f(x) = $ _____

b) $\lim\limits_{x \to +\infty} f(x) = $ _____

c) $\lim\limits_{x \to 1^-} f(x) = $ _____

d) $\lim\limits_{x \to 1^+} f(x) = $ _____

**Question 6**

Investigate the given graphs of three functions; one function is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify and match each curve.

(2)



| Position function | a | b | c |
|---|---|---|---|
| Velocity function | a | b | c |
| Acceleration function | a | b | c |

Give a suitable reason for you answer:

**Question 7**

Calculate the limit: (2)

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3}$$

**Question 8**

Can l'Hôspital's Rule be used to calculate the following limit? Give a reason for your answer.

(2)

$$\lim_{x \to \pi^-} \frac{sinx}{1 - cosx}$$

**Question 9**

Find $f(x)$ given: (2)

$$f'(x) = 2e^x + secxtanx$$

**Question 10**

Find the integral: (2)

$$\int_0^1 (y^3 + 1.8y^2)dy$$

**SECTION B2: APPLICATION QUESTIONS** **[Total 29]**
**IN THIS SECTION, SHOW <u>ALL THE STEPS</u> TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.**
**USE THE BLANK PAGES (ON THE OPPOSITE SIDE) FOR ROUGH WORK.**
_____

**Question 11**
Find the derivative of the function using the definition of the derivative: (4)

$$g(x) = \sqrt{4 - x}$$

## Question 12

Given the function:

$$f(x) = \frac{1}{x^2 - 9}$$

12.1    Can you find any horizontal asymptotes for the function? Show calculations.                    (2)

12.2    Can you find any vertical asymptotes for the function? Show calculations.                    (2)

12.3    Find the intervals over which the function is increasing and/or decreasing. Show all
        calculations.                    (3)

12.4    Show that $f''(x) = \frac{6(x^2+3)}{(x^2-9)^3}$                                                    (3)

12.5    Hence, interpret the second derivative and find the intervals over which the graph is concave upwards and/or concave downwards.                                    (3)

12.6    Can you find any inflection points? Motivate your answer.                      (1)

## Question 13

Find the equation of the tangent line to the curve $y = \frac{e^x}{(1+x^2)}$ at the point $\left(1, \frac{e}{2}\right)$. (3)

## Question 14

Find the integral:

14.1    $\int (csc^2 t - 2e^t)dt$ (2)

14.2    $\int \frac{sin2x}{sinx} dx$                                                                                                    (2)

## Question 15
The acceleration function (in $m/s^2$) and the initial velocity are given for a particle moving along a line:

$$a(t) = t + 4, \qquad v(0) = 5, \qquad 0 \leq t \leq 10$$

Find:

15.1    The velocity at time $t$.                                                                                      (2)

15.2    The distance travelled during the given time interval.                                          (2)

**USE THIS PAGE TO RE-DO ANY QUESTION YOU MAY HAVE CANCELLED.**

**TOTAL OF 100 POSSIBLE MARKS ON PAPER**