



FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS
CONSTRUCTION ENGINEERING

MODULE: MATEBD1
CAMPUS: DFC
ASSESSMENT: DEFERRED EXAMINATION

ASSESSOR:
MODERATOR:

DR SR HERBST
MR MORAPELI

MARKS OBTAINED	<u>120</u>	%
-----------------------	-------------------	----------

DURATION: 180 MINUTES

MARKS: 120 + 10 BONUS MARKS

INITIALS AND SURNAME: _____
STUDENT NUMBER: _____
CONTACT NUMBER: _____

INSTRUCTIONS: WRITE YOUR STUDENT NUMBER AND PARTICULARS IN THE SPACE PROVIDED
ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET
USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE)

REQUIREMENTS: INFORMATION BOOKLET (AS ISSUED TO YOU IN THE TEST)
NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

SECTION A: DIFFERENTIATION [55 MARKS]**INSTRUCTIONS**

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.

1. Compute the derivatives of the following expressions

a. $(3x^2 + \sin 2x - e^{-5x} + \frac{2}{x} - 4)$ (5)

b. $\sin(x^3 - x)$ (5)

c. $\tan^2 t$ (5)

2. Given the functions $x(t)$ and $y(t)$ find the derivative $\frac{dy}{dx}$.

a. $x(t) = a \cos^3 t, y(t) = a \sin^3 t$ (5)

b. $x(t) = \operatorname{sech} t, y(t) = t - \tanh t$ (7)

3. Compute the derivative of the following expressions or functions

a. $y^2 = x(x - 1)^2$ (5)

b. $y^2 = e^{\sin x} + \arccos(\ln x)$ (8)

4. Compute the rate of change of the following functions

a. $f = \sqrt{x^2 + (y + z)^2}$ with $x'(t) = y'(t) = 1$ and $z'(t) = -1$ (10)

SECTION B: INTEGRATION [65 MARKS]**INSTRUCTIONS**

SHOW ONLY THE MOST IMPORTANT STEPS EXCEPT WHERE ASKED OTHERWISE. PERFORM ROUGH CALCULATIONS ON BLANK PAPER.

1. Compute the integrals, show full workings:

a. $\int \frac{1}{25} x^5 + 2 \cos 3x - 3 \ln(x^{\frac{1}{5}}) + \sinh(2x - 1) dx$ (5)

b. $\int_1^{e^\pi} \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} - \frac{\cos(2 \ln \sqrt{x})}{x}$ (10)

c. $\int \tanh \theta d\theta$ (Show full workings) (5)

2. Integrate the following functions using an appropriate substitution

a. $\sin^5 x$ (3)

b. $\sin^4 t \cos^2 t$ (7)

c. $\frac{\sin x}{9 + \cos^2 x}$ (5)

d. $\frac{x-3}{25-x^2}$ (8)

e. $\frac{1-\sin x}{1+\sin x}$ (7)

3. Find the volume of the following:

- a. A portion of a cylinder where $1 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{3}, 0 \leq z \leq 1$. Hint: compute the triple integral $\iiint r \, dr d\theta dz$ over the appropriate bounds. (5)

4. Find the root mean square of the function $f = \sqrt{r}e^{-r^2}$ between 0 and 5. (10)

SECTION C: MATRICES [10 BONUS MARKS]

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.

1. Find the eigenvalues of the matrix $\begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix}$ (5)
2. Find the corresponding eigenvectors of $\begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix}$. (5)