UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

CONSTRUCTION ENGINEERING

MODULE: **MATEBD1**

CAMPUS: **DFC**

ASSESSMENT: **SEMESTER 2, FINAL EXAM**

DATE: **09 NOVEMBER 2019** **TIME: 08:00-11:00**

DURATION: **180 MINUTES**

ASSESSOR: **DR SR HERBST**
MODERATOR: **DR T PAEPAE**

MARKS OBTAINED	$\frac{\quad}{120}$	**%**

DURATION: **180 MINUTES** **MARKS: 120 + 10 BONUS MARKS**

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

INSTRUCTIONS: **WRITE YOUR STUDENT NUMBER AND PARTICULARS IN THE SPACE PROVIDED**
ANSWER ALL QUESTIONS IN THE ANSWER BOOKLET
USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE)

REQUIREMENTS: **INFORMATION BOOKLET (AS ISSUED TO YOU IN THE TEST)**
NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

SECTION A: DIFFERENTIATION [55 MARKS]

INSTRUCTIONS

SHOW ONLY THE MOST IMPORTANT STEPS EXCEPT WHERE ASKED OTHERWISE. PERFORM ROUGH CALCULATIONS ON BLANK PAPER.

1. Calculate the derivative of the following functions, show full workings:

   a. $y = \frac{1}{25}x^5 + 2\cos 3x - 3\ln(x^{\frac{1}{5}}) + \sinh(2x - 1)$ (5)

   b. $f = \sin 3t \cosh(4 \sin 3t)$ (5)

   c. $y = \sec\sqrt{x} - \cos(2x^4)$ (5)

d. $g = \tanh\theta$ (Show full workings) (5)

e. $y = e^{2\sin^2 x} - \arctan\sqrt{x^2 - 1}$ (10)

f. $y = \operatorname{arcsec} x + \arcsin e^{2x}$ (10)

2. Consider the two functions $x(t),\ y(t)$ and determine the derivative $\frac{dy}{dx}$.

a. $x(t) = t - \sin t\,,\ y(t) = 1 - \cos t$ (3)

b. $x(t) = 2\cos t + \cos 2t\,,\ y(t) = 2\sin t - \sin 2t$. *Hint:* $\frac{1-\cos t}{\sin t} = \tan\frac{t}{2}$. (7)

3. Find the rate of change $\frac{df}{dt}$, for the function $f = \sqrt{x^2 + y^2 + z^2}$. (5)

SECTION B: INTEGRATION [55 MARKS]

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.

4. Compute the following integrals

   a. $\int (3x^2 + \sin 2x - e^{-5x} + \frac{2}{x} - 4)dx$ (5)

b. $\int (3x^2 - 1)\sin(x^3 - x)dx$ (5)

c. $\int_0^{\pi/4} \tan^2 t \ dt$ (5)

d. $\int_0^{\pi} \sin^3\theta \cos^2\theta \ d\theta$ (10)

e. $\int_{-1}^{1} \frac{5}{(\theta^2+\theta-6)} \ d\theta$ (7)

f. $\int_0^{0.5} -\frac{2}{\cos 2\theta}\ d\theta$ (8)

g. $\int_{-0.5}^{0.5} \frac{1}{1-x^4}\ dx$ (10)

h. $\int_0^{1.54} \frac{1}{1-\sin x}\, dx$ (5)

SECTION C: MATRICES [10 MARKS]

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE.

1. Find the eigenvalues of the matrix $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$ (5)

2. Find the corresponding eigenvectors of $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$. (5)

SECTION D: BONUS [10 MARKS]

1. Consider the vector variable $\vec{U} = \begin{pmatrix} \rho \\ p \end{pmatrix}$ and the vector function $\vec{F} = \begin{pmatrix} p \\ \frac{p^2}{\rho} + \rho\, c^2 \end{pmatrix}$.

   Calculate the Jacobian matrix and find the corresponding eigenvalues. *In your final answer, let* $p = \rho u$ *then simplify.* (10)

End of assessment – Total 130 marks available

Use this space if you want to redo any question(s). Please indicate clearly at the relevant question(s) that the solution is on this page.