UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	MATENB1/MATE0B1 APPLICATIONS OF CALCULUS FOR ENGINEERS
CAMPUS	APK
ASSESSMENT	EXAMINATION

DATE 09/11/2019	TIME 08:30
ASSESSOR(S)	DR K SEBOGODI MR T MUDZIIRI SHUMBA
INTERNAL MODERATOR	DR A CRAIG
DURATION 120 MINUTES	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 13 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [11 marks]

For questions 1.1 – 1.10, choose the correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Determine whether the following statement is true or false: The partial fraction decomposition of $\dfrac{x^2+4}{x^2(x^2-4)}$ is $\dfrac{A}{x^2}+\dfrac{B}{x-2}+\dfrac{C}{x+2}$. (1)

(a) True

(b) False

1.2 If $\displaystyle\int_a^\infty f(x)\,dx$ and $\displaystyle\int_a^\infty g(x)\,dx$ are both divergent, then $\displaystyle\int_a^\infty [f(x)+g(x)]\,dx$ is also divergent. (1)

(a) True

(b) False

1.3 Let $f(x)=1-x^{\frac{2}{3}}$. Which one of the following statements is correct? (1)

(a) f is continuous on $[-1,1]$.

(b) f is differentiable on $(-1,1)$.

(c) $f(-1)\neq f(1)$.

(d) f satisfies Rolle's Theorem with $a=-1$ and $b=1$.

(e) None of the above.

1.4 A number c is a critical number of a function f. Which one of the following statements is false? (1)

(a) The number c is in the domain of f.

(b) The derivative of f may take the value 0 at c.

(c) The number c may lie outside the domain of f.

(d) The derivative of f may not exist at c.

(e) None of the above.

1.5 Find the number a such that the average value of the function $f(x) = e^x$ on the interval $[0, a]$ is equal to $\dfrac{1}{\ln 2}$. (1)

(a) $\dfrac{1}{\ln 2}$
(b) $\ln 2$
(c) 2
(d) e^2
(e) None of the above.

1.6 Which of the following integrals gives the area of the surface by rotating the curve $y = \sqrt{x},\ (0 \leqslant x \leqslant 4)$ about the x-axis? (1)

(a) $\displaystyle\int_0^4 2\pi x\sqrt{1+\frac{1}{2x}}\,dx$
(b) $\displaystyle\int_0^4 2\pi x\sqrt{1+\frac{1}{4x}}\,dx$
(c) $\displaystyle\int_0^4 2\pi\sqrt{x}\sqrt{1+\frac{1}{2x}}\,dx$
(d) $\displaystyle\int_0^4 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx$
(e) None of the above.

1.7 What is the coefficient of x^3 in the expansion $(2+x)^5$? (1)

(a) 40
(b) 20
(c) 10
(d) 80
(e) None of the above.

1.8 Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(18, 0)$. (1)

(a) $\left(\dfrac{35}{2}, \sqrt{\dfrac{35}{2}}\right)$
(b) $\left(\dfrac{37}{2}, \sqrt{\dfrac{37}{2}}\right)$
(c) $\left(\dfrac{35}{2}, \sqrt{\dfrac{37}{2}}\right)$
(d) $\left(\sqrt{\dfrac{37}{2}}, \dfrac{35}{2}\right)$
(e) $\left(\sqrt{\dfrac{35}{2}}, \dfrac{35}{2}\right)$

1.9 Find the corresponding rectangular equation for the curve represented by the parametric equations $x = 7 + \frac{2}{t}$, $y = t - 9$ by eliminating the parameter. (1)

(a) $y = \frac{x-7}{2} - 9$

(b) $y = \frac{2}{x+7} + 9$

(c) $y = \frac{2}{x-7} - 9$

(d) $y = \frac{2}{x-7} + 9$

(e) $y = \frac{2}{x+7} - 9$

1.10 At which value of x does the inflection point of the function $f(x) = xe^{-5x}$ occur? (2)

(a) $\frac{1}{5}$

(b) 5

(c) $\frac{2}{5}$

(d) 0

(e) None of the above.

Question 2 [2 marks]

Let $z = x^3y^2$, where both x and y are changing with time. At a certain instant when $x = 1$ and $y = 2, x$ is decreasing at the rate of 2 units/s, and y is increasing at the rate of 3 units/s. How fast is z changing at this instant?

Question 3 [5 marks]

Verify that the function $f(x) = x(x-1)(x-2)$ satisfies the hypotheses of the mean value theorem on $\left[0, \frac{1}{2}\right]$. Find the number c such that the condition of the conclusion of the theorem is satisfied.

Question 4 [14 marks]

Evaluate the following integrals:

(a) $\int \cos(\ln x)\, dx$ (4)

(b) $\int \cot^5 \phi \csc^3 \phi \, d\phi$ (3)

(c) $\int_0^1 \frac{1}{(x^2+1)^2} \, dx$ (4)

(d) $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$ (3)

Question 5 [4 marks]

Find the length of the curve $y = \displaystyle\int_1^x \sqrt{\sqrt{t}-1}\,dt$ for $1 \leqslant x \leqslant 16$. (4)

Question 6 [3 marks]

Sketch a graph of a function f that satisfies the following conditions:

(a) $\lim_{x\to-\infty} f(x) = 1$ and $\lim_{x\to\infty} f(x) = -1$

(b) $\lim_{x\to-1^-} = -\infty$ and $\lim_{x\to-1^+} = \infty$

(c) $f'(x) > 0$ for $x < -2$

(d) $f'(x) = 0$ for $x = -2$

Question 7 [4 marks]

Consider the region R bounded by $y = \sqrt{x-1}$ and $y = x - 1$.

(a) Sketch the region R, indicating the points of intersection. (1)

(b) Use the **Disk/Washer Method** to find the volume of the solid obtained by rotating the region R about the line $x = 2$. (3)

Question 8 [6 marks]

(a) Solve the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 2 \cos^3 x \sin x - 1, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Write your solution in the form $y = f(x)$. (3)

(b) Solve the initial value problem: (3)

$$\frac{dy}{dx} = \frac{-15}{y^2 \cos^2 x}, \qquad y(0) = 3$$

Question 9 [6 marks]

(a) Consider the parametric equations below:

$$x = t\cos t, \qquad y = t\sin t.$$

Find an equation of the tangent to the curve at the point $t = 5\pi$. (3)

(b) Sketch the graph of the polar equation (3)

$$r = 2\sin\theta.$$

Question 10 [5 marks]

Consider the matrices below and then write down the following:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) $A + B$ (1)

(b) A^T (1)

(c) AB (2)

(d) $\text{trace}(A)$ (1)

Question 11 [3 marks]

Consider the system of equations below:

$$\begin{aligned} 5x - 5y - 15z &= 40 \\ 4x - 2y - 6z &= 19 \\ 3x - 6y - 17z &= 41 \end{aligned}$$

Represent the system of equations as an augmented matrix and then use elementary row operations to write the augmented matrix in reduced row echelon form and write down the solutions to the equations. (3)

Question 12 [3 marks]

Use the Binomial Theorem to expand $\left(\frac{1}{x} - x^2\right)^5$. Simplify all coefficients. (3)

Question 13 [4 marks]

(a) State the Extreme Value Theorem. (2)

(b) State the Mean Value Theorem. (2)