## FACULTY OF SCIENCE

| DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS |  |  |
| :---: | :---: | :---: |
| MODULE: MATH | ATICS FOR FINANCE AN |  |
| CAMPUS: APK/ |  |  |
| ASSESSMENT: SUPP | ENTARY EXAM |  |
| DATE: | JANUARY 2020 |  |
| ASSESSORS: | MS M JUGA <br> MR T MOHUBEDU <br> MR MSW POTGIETER |  |
| INTERNAL MODERATOR: | DR. J MBA | 76 |
| DURATION: | 2 HOURS |  |

INITIALS AND SURNAME: $\qquad$

STUDENT NUMBER:

CONTACT NUMBER:

## NUMBER OF PAGES: 15 (INCLUDING COVER PAGE)

INSTRUCTIONS:

- ANSWER ALL THE QUESTIONS IN PEN.
- ALL GRAPHS MUST BE DRAWN IN PEN.
- NO PENCIL OR TIPEX ALLOWED.
- SHOW ALL THE NECESSARY CALCULATIONS CLEARLY.
- IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM
- ONLY SCIENTIFIC CALCULATORS ARE ALLOWED.
- IF NECESSARY, ROUND OFF TO TWO DECIMAL PLACES.
- THE QUESTIONS CAN BE ANSWERED IN ANY ORDER.


## Question 1

1.1 Maxi needs to save up R 15,000.00 in three years' time for school. Determine her deposit if her accounts earn 9\% per year, compounded continuously.
1.2 Tsidi would like to open a savings account and approaches several banks. The three interest rate options she is given are:
A, $4.1 \%$ per year compounded daily.
B, $4.15 \%$ per year compounded weekly.
C, $4.19 \%$ per year compounded monthly.
Calculate the effective rate for each option and select the best option for Tsidi.

### 1.3 How long will it take to triple an investment subjected to an interest rate of $9.55 \%$ per year compounded semi-annually.

## Question 2

Janet is saving up to purchase a local business. When she had R 150,000.00 saved the business was put up for sale. The owner accepted Janet's offer which was $85 \%$ of $R$ $500,000.00$ price. Her savings is not enough and decides to take out a loan to complete the purchase. The loan is to be repaid with monthly payments over 15 years and the loan is subjected to an interest rate of $9 \%$ per year compounded monthly.

Determine:
2.1 The required monthly payment.
2.2 The principle outstanding after the $23^{\text {rd }}$ payment.
2.3 The principle outstanding at the beginning of $16^{\text {th }}$ month.

2.4 The interest repaid in the $16^{\text {th }}$ month.
2.5 The principle repaid with the $16^{\text {th }}$ payment.
[2]
2.6 The finance charge.

## Question 3

You have to repay a debt of $\mathrm{R} 4,000.00$ in due in 5 years' time and a debt of $R$ $8,000.00$ due int 8 years' time. Instead you make a payment of R 2,000.00 now and a final payment after 10 years.
3.1 Sketch a timeline, indicating all debts, payment and years.
3.2 Calculate the value of the final payment.

## Question 4

Surefire LLC will need to replace a milling machine in 5 years' time. To do this the company starts a sinking fund making monthly contributions of $\mathrm{R} 10,000.00$. The machine will cost R 1,500,000.00 in 5 years' time. If the interest rate is $9 \%$ per year, will they have sufficient funds to purchase the new machine. Motivate your answer.

## Question5

Differentiate the following, but do not simplify your answer:
$5.1 \quad y=e^{x}+\ln x-4 x^{2}$
$5.2 y=\left(2 x^{5}-4 x^{2}\right)\left(3 x^{-3}+5 x^{3}\right)$
$5.3 y=\frac{5 x^{4}-2 x}{2 x^{4}-4 x^{3}}$
$5.4 \quad f(x)=\left(e^{x}-4 x^{5}\right)^{50}$

## Question 6

Find the fourth derivative, $y^{(4)}$, of the given function:

$$
y=e^{4 x}
$$

## Question 7

Given the following Average cost function:

$$
A C=2,000 q^{4}+\frac{2,000}{q}
$$

7.1 Determine the total cost function when $q=500$.

### 7.2 Determine the marginal cost function.

7.3 If the production quantity is to be increased by 1 unit at a present production level of 1000 units, determine by how much the total cost would increase

## Question 8

In preparation for the 2019 Rugby World Cup, the Springboks reviewed previous games played against the All Blacks. The scores for the last 13 games played are presented below:

$$
7,7,5,5,5,0,10,15,18,17,5,8,7
$$

8.1 Complete the following grouped frequency table using the given information above.

| Class | Count | Frequency | Relative Frequency |
| :---: | :---: | :---: | :---: |
| $1-5$ |  |  |  |
| $6-10$ |  |  |  |
| $11-15$ |  |  |  |
| $16-20$ |  |  |  |
| TOTAL |  |  |  |

8.2 Determine the following parameters using the information presented in Table 1:

| Mean |  |
| :--- | :--- |
| Mode |  |
| Median |  |

## Question 9

Suppose a coin is tossed three times with the orientation of the coin being recorded when it lands. Then:

$$
S=\{T T T, T T H, T H T, H T T, T H H, H T H, H H T, H H H\}
$$

is the sample space of the experiment with events:

$$
A=\{T T H, T H T, H H T, T T T, H H H\} \text { and } B=\{H H T, H T H, T H H\} .
$$

### 9.1 Determine $P(B)$.

9.2 Determine $P(A \cap B)$.
9.3 Determine $P(A \cup B)$.
9.4 Determine $P(B \mid A)$.
9.5 Are events $A$ and $B$ mutually exclusive? Motivate your answer.

## Question 10

Given the following probabilities:

$$
P(A)=0.3, P(A \cup B)=0.9 \text { and } P(B)=0.6
$$

10.1 Determine the probability of A or $\mathrm{B}, P(A \cap B)$.
10.2 Are events $A$ and $B$ mutually exclusive? (YES or NO). Motivate your answer.

## Question 11

On the provided Venn Diagram shade the region that represents:
11.1 $S \cap M \cap P$

11.2 $M \cap P$


## Question 12

12.1 If $X \sim N(50,49)$, use Annexure B to determine $P(X>46)$.
12.2 If $X \sim N(100,225)$, use Annexure B to determine $P(96<X<104)$.

## ANNEXURE A

| $R \frac{r}{m}\left[\frac{1-\left(1+\frac{r}{m}\right)^{-n m+k-1}}{\frac{r}{m}}\right]$ | $R\left[1-\frac{r}{m} \times \frac{1-\left(1+\frac{r}{m}\right)^{-n m+k-1}}{\frac{r}{m}}\right]$ |  |
| :---: | :---: | :---: |
| $n m R-A$ | $R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-n m}}{\frac{r}{m}}\right]$ | $R\left[\frac{1-\left(1+\frac{r}{m}\right)^{-n m+k-1}}{\frac{r}{m}}\right]$ |



## ANNEXURE B

|  | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 1}$ | $\mathbf{0 . 0 2}$ | $\mathbf{0 . 0 3}$ | $\mathbf{0 . 0 4}$ | $\mathbf{0 . 0 5}$ | $\mathbf{0 . 0 6}$ | $\mathbf{0 . 0 7}$ | $\mathbf{0 . 0 8}$ | $\mathbf{0 . 0 9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{0 . 0}$ | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| $\mathbf{0 . 1}$ | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| $\mathbf{0 . 2}$ | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| $\mathbf{0 . 3}$ | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| $\mathbf{0 . 4}$ | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| $\mathbf{0 . 5}$ | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| $\mathbf{0 . 6}$ | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| $\mathbf{0 . 7}$ | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| $\mathbf{0 . 8}$ | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| $\mathbf{0 . 9}$ | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| $\mathbf{1 . 0}$ | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| $\mathbf{1 . 1}$ | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| $\mathbf{1 . 2}$ | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| $\mathbf{1 . 3}$ | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| $\mathbf{1 . 4}$ | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| $\mathbf{1 . 5}$ | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| $\mathbf{1 . 6}$ | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| $\mathbf{1 . 7}$ | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| $\mathbf{1 . 8}$ | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| $\mathbf{1 . 9}$ | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| $\mathbf{2 . 0}$ | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |


| $\mathbf{2 . 1}$ | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{2 . 2}$ | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| $\mathbf{2 . 3}$ | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| $\mathbf{2 . 4}$ | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| $\mathbf{2 . 5}$ | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| $\mathbf{2 . 6}$ | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| $\mathbf{2 . 7}$ | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| $\mathbf{2 . 8}$ | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| $\mathbf{2 . 9}$ | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| $\mathbf{3 . 0}$ | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

Use this space if you want to redo a question. Clearly indicate at the question that the answer is on Page 14/15.

