UNIVERSITY OF JOHANNESBURG

DEPARTMENT OF MATHEMATICS

COURSE: MAT8X06 (THEORY)

NOVEMBER EXAMINATION

DATE: NOVEMBER / DECEMBER 2019

TIME: 3 HOURS **MARKS: 50**

Examiners: Dr. E. J. Joubert
External Examiner: Dr. S. Dorfling

Student number: _____

Surname and initials: _____

Contact tel. number: _____

PUNTE	%

1. This paper consists of 2 pages.
2. Answer each question in the provided booklet.
3. Show all your calculations and arguments clearly.

QUESTION 1 **[15]**

1.1) Let G be a connected, plane graph with n vertices, m edges and r regions. Prove that $n-m+r = 2$. [5]

1.2) Let G be a planar, bipartite graph of order n and size m, where $n \geq 3$. Prove that $m \leq 2n-4$.[5]

1.3) Show that $cr(K_{2,2,3}) = 2$. [5]

QUESTION 2 **[11]**

2.1) Let S be a dominating set of G. Prove the following: S is a minimal dominating set if and only if for every vertex $v \in S$ at least one of the following holds: [6]

i) $\exists w \in V(G) - S : N(w) \cap S = \{v\}$.
ii) v has no neighbour in S

2.2) Let G be a graph of order n without isolated vertices. Prove that $\gamma(G) \leq \frac{n}{2}$. [5]

QUESTION 3 **[6]**

3.1) A graph G is $2-$factorable if and only if G is $2k-$regular for some k. [6]

QUESTION 4 **[18]**

4.1) Prove the following theorem: Every graph of order $n \geq 6$ with at least $3n - 5$ edges contains two vertex-disjoint cycles. [10]

4.2) State and prove Turàn's theorem. [8]

TOTAL: 50