UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

	Examiner	Moderator
Paper 1 30 Marks		
Paper 2 70 Marks		
EM/100		

	Examiner	Moderator
SM		
EM		
FM		

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

N DIP: *MECHANICAL AND INDUSTRIAL ENGINEERING*

MODULE: **ENGINEERING MATHEMATICS 3 (MAT3AW3)**

CAMPUS: **DFC**

ASSESSMENT: **MAIN EXAMINATION**

(PAPER 2)

DATE: 14 NOVEMBER 2019 **SESSION: 08:30-11:30**

ASSESSOR: **DR SM SIMELANE**

MODERATOR: **DR T MASEBE**

DURATION: 180 MINUTES **MARKS: 100**

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: **18 PAGES (INCLUDING COVER PAGE)**

INSTRUCTIONS: **WRITE YOUR STUDENT NUMBER AND PARTICULARS IN THE SPACE PROVIDED.**
ANSWER ALL QUESTIONS IN THE SPACES PROVIDED.
USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE).

THIS EXAM IS ASSESSING SKILLS FOR SOLVING WELL-DEFINED ENGINEERING PROBLEMS AS OUTLINED IN ELO 1.

REQUIREMENTS: **INFORMATION BOOKLET (AS ISSUED TO YOU IN THE EXAM)**

Question 1

Determine the following

a. $L\{2\ t^3 \cosh 2\ t\}$ (3)

b. $L^{-1}\left\{\frac{1}{e^p(p^2-p)}\right\}$ (4)

c. $\frac{2}{D^2-9}\{\sinh 2t\}$ (4)

d. $\frac{1}{D^2+D-5}\{2t^2\}$ (4)

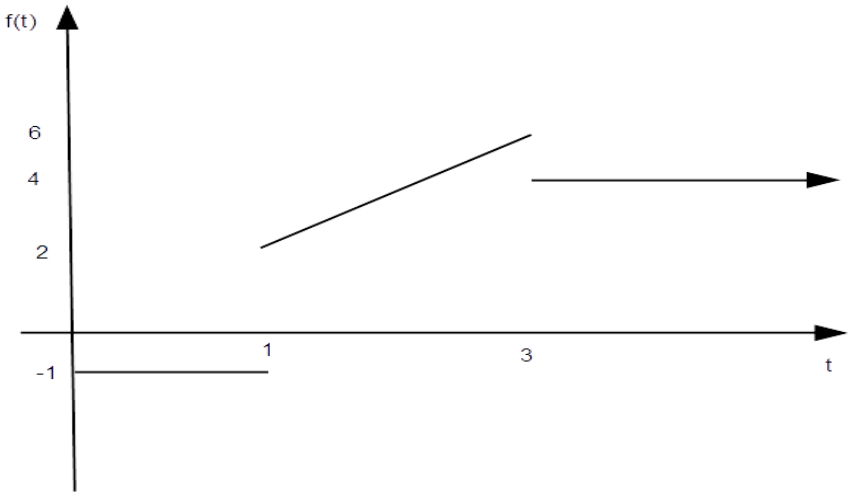
e. $\int_0^1 \cos\frac{n\pi x}{2}\, dx$ (4)

f. Is the function $g(t) = \begin{cases} t-3, & -2 \le t < 0 \\ t+3, & 0 \le t < 2 \end{cases}$ odd, even or neither? (2)

Show how you get to your conclusion.

Question 2

Given the function $f(t)$ as



a. Express the given function $f(t)$ in unit step functions. (3)

b. Express the function $f(t)$ found above in Heaviside form. **Fully simplify.** (2)

c. Compute the Laplace transforms of $f(t)$. (4)

Question 3

Determine the unique solutions of the following differential equations by using the **Laplace Transforms**, subject to the indicated initial conditions.

a. $y'' + 4y = 5$, subject to $y(0) = 0$ and $y'(0) = 3$. (8)

b. $x''(t) + 4\,x'(t) + 6\,x(t) = 0$ subject to $x(0) = 3$ and $x'(0) = 2$. (6)

Question 4

a. The mathematical model of an arbitrary system is described by the following differential equation

$$\theta'' + 6\,\theta' + 9\,\theta = t\,e^{-3t}\ .$$

Using **Laplace Transforms**, determine the motion $\theta(t)$ if the initial displacement is $\theta(0) = 6$ and the initial velocity applied to the mass is $\theta'(0) = 2$. Will the system ever come to rest? (6)

b. Use **D-Operator Methods** to solve the differential equation for $\theta(t)$ and use the given initial conditions to determine the particular solution. (8)

Question 5

Use **D-Operator Methods** to solve

a. $(D^2 - 3D + 2)y = e^t \sin t$ (8)

b. $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^t$ (7)

c. Solve for ***x*** only

$$\frac{dx}{dt} - 4x + y = 1$$
$$\frac{dy}{dt} + 6y \ = t^2 e^t$$

(10)

Question 6

a. Find a Fourier series for the following function

$$f(x) = \begin{cases} 2x & -4 \le x < 0 \\ 0 & 0 \le x < 4 \end{cases}; \qquad f(x) = f(x+8) \qquad (13)$$

b. The table below defines a discrete function *f(x)* of period 2π. Determine the Fourier series of *f(x)* up to the second harmonic. (Work to 2 decimal places) (6)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
$f(x)$	0.48	1.78	2.22	-0.48	-1.78	-2.22

End of assessment – Total 102 marks available

Use this space if you want to redo any question(s). Please indicate clearly at the relevant question(s) that the solution is on this page.