UNIVERSITY OF JOHANNESBURG

SM	
EM	
FM	

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

NATIONAL DIPLOMA:
ELECTRICAL/MECHANICAL/INDUSTRIAL/MINING ENGINEERING, MINERAL SURVEYING AND ANALYTICAL CHEMISTRY

MODULE: **MAT2AW2**
ENGINEERING MATHEMATICS 2

CAMPUS: **DFC**

JANUARY EXAMINATION

DURATION: **3 HOURS** **MARKS:** **100**

ASSESSOR: **VL SIXABA**
MODERATOR: MP SELOANE

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: 19
INSTRUCTIONS: ANSWER ALL QUESTIONS IN THE SPACES PROVIDED.
USE THE BACK OF EACH PAGE FOR ROUGH WORK USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE).

REQUIREMENTS: **NON PROGRAMMABLE CALCULATORS.**
FORMULA BOOKLET (PROVIDED).

SECTION A [20 MARKS]

INSTRUCTIONS

GIVE ONLY THE FINAL SIMPLIFIED ANSWER (CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE) IN THE SPACE PROVIDED

1. Find $\frac{dy}{dx}$ if $y = cot(cot^{-1}x)$ (2)

2. Find $\frac{dy}{dx}$ if $y = ln(x^2 + y^2)$. (2)

3. Find $\frac{d^2y}{dx^2}$ if $x = e^{-t}$ and $\frac{dy}{dx} = -e^{8t} - 7te^{8t}$ (2)

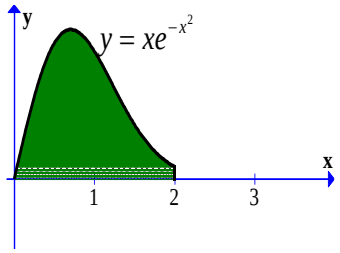
4. Find $\frac{\partial z}{\partial x}$ at the point (2,8), if $z = \sqrt{4x + y}$. (2)

5. Evaluate $\int \frac{sin(6x)}{1+cos(6x)} dx$. (2)

6. Evaluate $\int x5^x dx$. (2)

7. Evaluate $\int \frac{\sinh^{-1} x}{\sqrt{1+x^2}}\,dx$. (2)

8. Calculate the area of the region bounded by $y = xe^{-x^2}$, $x = 2$ and the $x = 0$ axis (2)
(see figure below)



9. Solve the differential equation: $\frac{dy}{dx} = y(1-y)$ (2)

10. Find the integrating factor for the equation: $xdy + (y - \cos x)dx = 0$ (2)

SECTION B [81 MARKS]

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWER CORRECT TO TWO DECIMAL PLACES WHERE APPLICABLE. SIMPLIFY YOUR ANSWERS FULLY.

11. Differentiate $y = x^{\ln x}$ (3)

12. Determine the slope $\frac{\mathrm{dy}}{\mathrm{dx}}$ of the curve $e^{\frac{x}{y}} = 5x - 2y$. (4)

13. If $z = 4sin(2x)cos(3y)$, determine

13.1. $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (2)

13.2. $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial^2 z}{\partial x \partial y}$ (4)

14. A projectile is fired from a position on the ground. The motion of the projectile in two dimensions is represented by the parametric equations $x = ln(t^2 + 1)$ and $y = t^3$:

14.1. Find an expression for the velocity $\frac{\mathrm{dy}}{\mathrm{dx}}$ of the projectile. (3)

14.2. Find an expression for the acceleration $\frac{d^2y}{\mathrm{dx}^2}$ of the projectile. (2)

15. The velocity μ of blood that flows in a blood vessel of length l and radius R is

$$\mu = \frac{PR^2}{4\eta l}$$

where P is the pressure of the blood vessel and η (constant) is the viscosity of blood. The application of the drug TPA has the effect of increasing the radius of the artery by 1,5%, decreaisng the length of the artery by 0,1% and decreasing the pressure by 2%. Find the percentage change in the velocity of blood flow (5)

16. When two resistors R_1 and $R_2$are connected in parallel, the total resistance is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 and R_2 are increasing at rates of $0{,}2\omega.s^{-1}$ and $0{,}01\omega.s^{-1}$ respectively, at what rate is R changing the instant when $R_1 = 12\omega$ and $R_2 = 68\omega$. (5)

17. Determine the following integrals

17.1. $\int 5tan^{-1}(2y)dy$ by using integration by parts. (5)

17.2. $\int_0^{\frac{\pi}{10}} \sqrt{1+\cos 5\theta}$. (6)

17.3. $\int \frac{-5e^x-9}{e^{2x}+4e^x+3}dx$. (5)

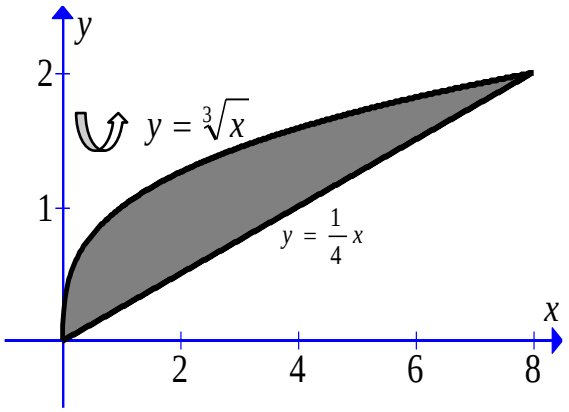
17.4. $\int \frac{cos(x)+sin(x)}{sin(2x)}dx$. (3)

17.5. $\int \sqrt{49 - x^2}dx$ by using trig substitution. (5)

17.6. $\int tan^6 x sec^4 x dx.$ (3)

18. Determine the **mean value** of the function $y = xe^{\frac{-x}{a}}$ from $x = 0$ to $x = a$ (4)

19. Determine the volume of the solid obtained by rotating the region enclosed by $y = \sqrt[3]{x}$ and $y = \frac{1}{4}x$ about the y –axis (5)



20. An apple pie with an initial temperature of 170℃ is removed from the oven and left to cool in a room with an air temperature of 20℃. The drop in temperature is modelled by

$$\frac{\mathrm{dT}}{\mathrm{dt}} = -r(T - T_e),$$

where T is the temperature of the object, T_e is the (constant) temperature of the environment and r is a constant of proportionality.

20.1. Find the particular solution of the model equation. (3)

20.2. Given that the temperature of the pie initially decreases at a rate of 3℃/min (i.e. $T'(0) = -3$℃/min). How long will it take for the pie to cool to a temperature of 30℃? (4)

21. Solve the following:

21.1. Bernoulli DE: $\frac{dy}{dx} + \frac{y}{x} = xy^2 sin(x)$ subject to $y(\pi) = 1$. (5)

21.2. Linear DE: $\frac{dy}{dx} + ytan(x) = sin(x) \quad if \;\; y(0) = 1$ (5)

End of assessment – Total 101 marks

Use this space if you want to redo any question(s). Please indicate clearly at the relevant question(s) that the solution is on this page.